International Journal of Pure and Applied Mathematics
Volume 120 No. 4 2018, 601-607
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v120i4.10
ijpam.eu

NEIGHBOURHOOD PRIME LABELING OF SOME SPECIAL GRAPHS

A. Delman[1], S. Koilraj[2], P. Lawrence Rozario Raj[3 §]
[1,2,3]PG and Research Department of Mathematics
St. Joseph's College
Tiruchirappalli, 620002, Tamilnadu, INDIA

Abstract: In this paper we investigate the neighbourhood prime labeling of an arbitrary super subdivision of helm, an arbitrary super subdivision of tadpole and and arbitrary super subdivision of triangular snake.

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for not defined here, we refer to Harary [2]. For standard terminology and notations related to number theory we refer to David [1] and graph labeling, we refer to Gallian [3]. The notion of prime labeling for graphs originated with Roger Entringer and was introduced in a paper by Tout et al [7]. The notion of neighbourhood prime labeling of graph was introduced by patel et al [5] and they presented the neighbourhood prime labeling of various graph. Sethuraman et al [6] introduced a new method of construction of graph called super subdivision graph and they proved arbitrary super subdivision of any path and cycle C_n are graceful. Lawrence et al. [4] presented arbitrary super subdivision of $(P_n \times P_m) \odot P_s$, tadpole graph and $C_n \odot P_m$ are signed product cordial graphs under some conditions.

Received: September 9, 2017
Revised: October 13, 2017
Published: March 12, 2018

[§]Correspondence author

2. Basic definitions

Definition 1. Let $G = (V, E)$ be a graph with n vertices. A function $f : V(G) \to \{1, 2, 3, ..., n\}$ is said to be a prime labeling, if it is bijective and for every pair of adjacent vertices u and v, $gcd(f(u), f(v)) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 2. Let $G = (V, E)$ be a graph with n vertices. A bijective function $f : V(G) \to \{1, 2, 3, ..., n\}$ is said to be a neighbourhood-prime labeling, if for every vertex $v \in V(G)$ with $deg(v) > 1, gcd\{f(u) : u \in N(v)\} = 1$. A graph which admits neighbourhood prime labeling is called a neighbourhood-prime graph.

Definition 3. Let G be a graph with q edges. A graph H is called a super subdivision of graph obtained from G by replacing every edge e_i of G by a complete bipartite graph K_{2,m_i} for some m_i, $1 \le i \le q$ in such a way that the end vertices of each e_i are identified with the two vertices of 2-vertices part of K_{2,m_i} after removing the edge e_i from graph G, if m_i is varying arbitrarily for each edge e_i then super subdivision is called arbitrary super subdivision of G.

Definition 4. A wheel graph W_n is a graph with $n + 1$ vertices, formed by connecting a single vertex to all the vertices of an n- cycle.

Definition 5. The helm H_n is the graph obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle.

Definition 6. Tadpole $T_{n,k}$ is a graph in which path P_k is attached to any one vertex of cycle C_n. The Tadpole $T_{n,k}$ has $n + k$ vertices and edges.

Definition 7. The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

The present work is focused on neighbourhood-prime labeling in the context of an arbitrary super subdivision of graphs. The neighbourhood-prime labeling of an arbitrary super subdivision of helm, tadpole and triangular snake are discussed.

3. Main results

Theorem 8. *Arbitrary super subdivision of Helm H_n is neighbourhood prime graph for $n \ge 3$.*

Proof. Let H_n be a Helm graph. Let v be the apex vertex, $u_1, u_2, ..., u_n$

be the vertices of degree four and $w_1, w_2, ..., w_n$ be the pendant vertices of G. Arbitrary super subdivision of H_n is obtained by replacing every edge of H_n with K_{2,m_i} and we denote this graph by G. Let $k = m_1 + m_2 + \cdots + m_{3n}$ and $v_{ij}(1 \leq i \leq n$ and $1 \leq j \leq m_i)$, $u_{ij}(1 \leq i \leq n$ and $1 \leq j \leq m_{n+i})$ and $w_{ij}(1 \leq i \leq n$ and $1 \leq j \leq m_{2n+i})$ be the vertices which are used for arbitrary super subdivision of H_n. Then $|V(G)| = k + 2n + 1$ and $|E(G)| = 2k$.

Define neighbourhood prime labeling $f : V(G) \to \{1, 2, \ldots, k + 2n + 1\}$ as follows, label the vertex v by 1, u_1 by 2, w_1 by 3 , label the vertices $u_2, ..., u_n$ by odd integers from 5 to $2n + 1$ and label the vertices $w_2, ..., w_n$ by even integers from 4 to $2n$. Also label the remaining vertices $v_{ij}(1 \leq i \leq n$ and $1 \leq j \leq m_i)$, $u_{ij}(1 \leq i \leq n$ and $1 \leq j \leq m_{n+i})$ and $w_{ij}(1 \leq i \leq n$ and $1 \leq j \leq m_{2n+i})$ by the remaining integers $2n + 2, 2n + 3, ..., k + 2n + 1$. Let x be any vertex of G and $deg(x) > 1$.

Case 1 : $x = v$

Since $f(u_2)$ and $f(u_3)$ are consecutive odd integers. The gcd of the labels of vertices in $N(v)$is 1.

Case 2 : $x = u_i$ for $1 \leq i \leq n$

The set $\{f(z) : z \in N(u_i), 1 \leq i \leq n\}$ contains 1. Therefore gcd of the labels of vertices in $N(u_i), 1 \leq i \leq n$ is 1.

Case 3 : $x = w_i$ for $1 \leq i \leq n$

The set $\{f(z) : z \in N(w_i), 1 \leq i \leq n\}$ contains atleast two consecutive integers. Therefore gcd of the labels of vertices in $N(w_i), 1 \leq i \leq n$ is 1.

Case 4 : $x = v_{ij}$ for $1 \leq i \leq n$ and $1 \leq j \leq m_i$

The set $\{f(z) : z \in N(v_{ij}), 1 \leq i \leq n$ and $1 \leq j \leq m_i\}$ contains the number 1. Therefore gcd of the labels of the vertices in $N(v_{ij})$, $1 \leq i \leq n$ and $1 \leq j \leq m_i$ is 1.

Case 5 : $x = u_{1,j}$ for $1 \leq j \leq m_{n+1}$

The set $\{f(z) : z \in N(u_{1j}), 1 \leq j \leq m_{n+1}\}$ contains the number 2 and 5. Therefore gcd of the labels of vertices in $N(u_{1,j})$, $1 \leq i \leq m_{n+1}$ is 1.

Case 6 : $x = v_{i,j}$ for $2 \leq j \leq n - 1$ and $1 \leq j \leq m_{n+i}$

The set $\{f(z) : z \in N(v_{ij}), 2 \leq i \leq n - 1$ and $1 \leq j \leq m_{n+i}\}$ contains only two consecutive odd integers $2i + 1$ and $2i + 3$. Therefore gcd of the labels of vertices in $N(v_{i,j})$, $2 \leq i \leq n - 1$ and $1 \leq j \leq m_{n+i}$ is 1.

Case 7 : $x = u_{n,j}$ for $1 \leq j \leq m_{2n}$

The set $\{f(z) : z \in N(u_{n,j}), 1 \leq j \leq m_{2n}\}$ contains $2n+1$ and 2. Therefore gcd of the labels of vertices in $N(u_{n,j}), 1 \leq j \leq m_{2n}$ is 1.

Case 8 : $x = w_{i,j}$ for $1 \leq i \leq n$ and $1 \leq j \leq m_{2n+i}$.

The set $\{f(z) : z \in N(w_{i,j}), 1 \leq i \leq n$ and $1 \leq j \leq m_{2n+i}$ contains atleast two consecutive integers. Therefore gcd of the labels of vertices in $N(w_{i,j})$,

$1 \leq i \leq n$ and $1 \leq j \leq m_{2n+i}$ is 1.

Hence f is a neighbourhood prime labeling. Thus an arbitrary super subdivision of Helm H_n is neighbourhood prime graph. □

Example 9. An arbitrary super subdivision of Helm H_4 and its neighbourhood prime labeling are shown in the Figure 1. Here $m_1 = 2, m_2 = 2, m_3 = 4, m_4 = 3, m_5 = 1, m_6 = 3, m_7 = 3, m_8 = 2, m_9 = 3, m_{10} = 2, m_{11} = 3$ and $m_{12} = 3$.

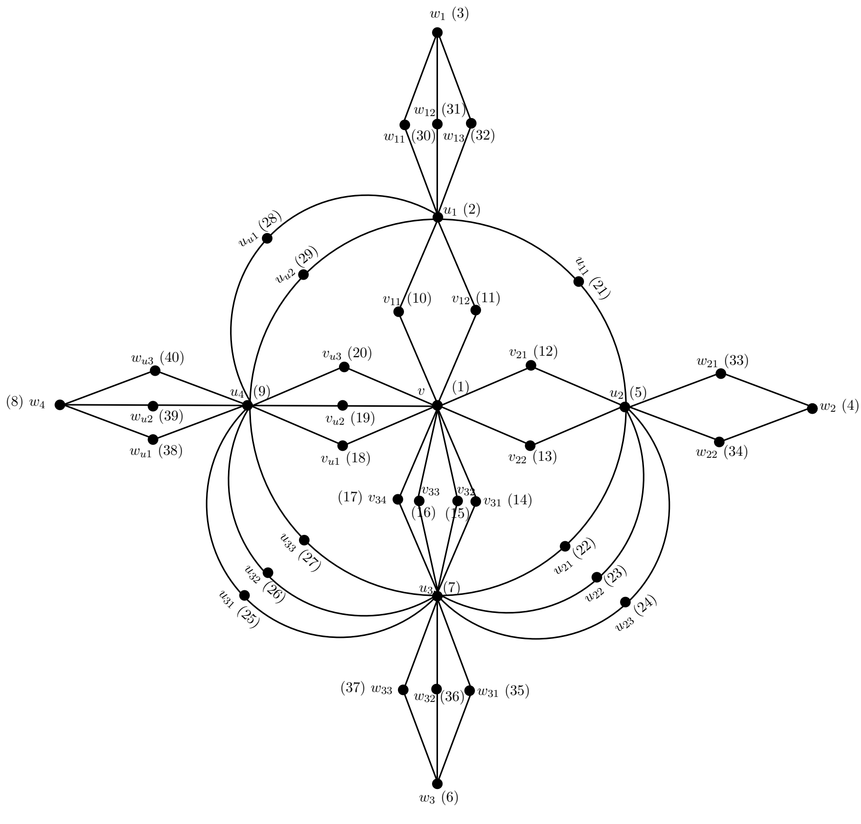

Figure 1.

Theorem 10. *Arbitrary super subdivision of any tadpole $T_{n.k}$ is neighbourhood prime graph.*

Proof. Let $T_{n,k}$ be a tadpole graph. Let $v_1, v_2, ..., v_n$ be the vertices of C_n and $v_{n+1}, v_{n+2}, ..., v_{n+k}$ be the vertices path P_k. Arbitrary super subdivision of $T_{n,k}$ is obtained by replacing every edge of $T_{n,k}$ with K_{2,m_i} and we denote this graph by G. Let $t = m_1 + m_2 + ... + m_{n+k}$ and $v_{i,j}(1 \leq i \leq n + k$ and

$1 \leq j \leq m_{n+k}$) be the vertices which are used for arbitrary super subdivision of $T_{n,k}$ and $v_{1,j}$ for $1 \leq j \leq m_1, v_{n,j}$ for $1 \leq j \leq m_n$ and $v_{(n+1),j}$ for $1 \leq j \leq m_{n+1}$ be the adjacent vertices of v_n. Then $|V(G)| = t + n + k$ and $|E(G)| = 2t$.

Define neighbourhood prime labeling $f : V(G) \to \{1, 2, ..., t + n + k\}$ as follows, label the vertices $v_1, v_2, ..., v_{n+k}$, $v_{i,j}(1 \leq i \leq n+k)$ and $1 \leq j \leq m_{n+k}$) by the integers form $1, 2, ..., t+n+k$. Let x be any vertex of G and $deg(x) > 1$.

Case 1 : $x = v_i$ for $1 \leq i \leq n + k$.

The set $\{f(z) : z \in N(v_i)$, for $1 \leq i \leq n + k\}$ contains atleast two consecutive integers. Therefore gcd of the labels of vertices in $N(v_i)$, for $1 \leq i \leq n+k$ is 1.

Case 2 : $x = v_{ij}$ for $2 \leq i \leq n + k$ and $1 \leq j \leq m_i$

The set $\{f(z) : z \in N(v_{i,j})$, for $2 \leq i \leq n + k$ and $1 \leq j \leq m_i\}$ contains exactly two consecutive integers. Therefore gcd of the lables of vertices in $N(v_{i,j})$, for $2 \leq i \leq n + k$ and $1 \leq j \leq m_i$ is 1

Case 3 : $x = v_{1,j}$ and $1 \leq j \leq m_1$

The set $\{f(z) : z \in N(v_{1,j})$, for $1 \leq j \leq m_1\}$ contains an integer 1. Therefore gcd of the labels of vertices in $N(v_{1,j})$, for $1 \leq j \leq m_1$ is 1.

Hence f is a neighbourhood prime labeling. Thus an arbitrary super subdivision of any tadpole $T_{n,k}$ is neighbourhood prime graph. □

Example 11. An arbitrary super subdivision of tadpole $T_{4,3}$ and its neighbourhood prime labeling are shown in Figure 2. Here $m_1 = 2, m_2 = 3, m_3 = 2, m_4 = 3, m_5 = 3, m_6 = 1$ and $m_7 = 2$.

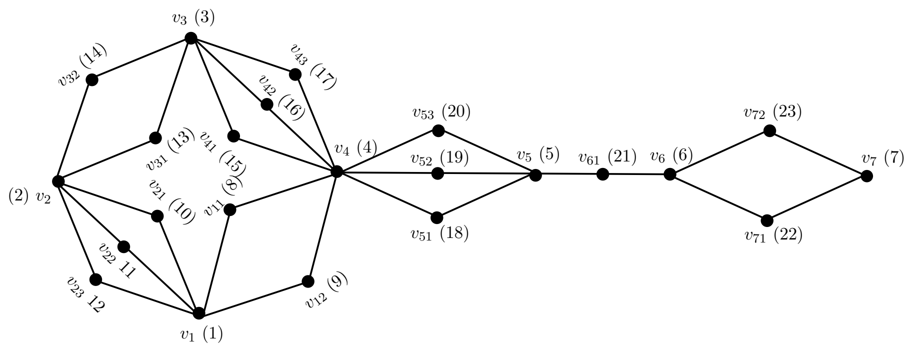

Figure 2

Theorem 12. *Arbitrary super subdivision of T_n is neighbourhood prime graph*

Proof. Let T_n be a triangular snake graph and $v_1, v_2, ..., v_n, w_1, w_2, ..., w_{n-1}$ be the vertices of T_n, where $v_1, v_2, ..., v_n$ be the vertices of path in T_n.

Arbitrary super subdivision of T_n is obtained by replacing every edge of T_n with K_{2,m_i} and we denote this graph by G. Let $t = m_1 + m_2 + ... + m_{3n-3}$. Let $v_{i,j}(1 \leq i \leq n-1$ and $1 \leq j \leq m_i)$ and $w_{i,j}(1 \leq i \leq 2n-2$ and $1 \leq j \leq m_{n-1+i})$ be the vertices which are used for arbitrary super subdivision of T_n, $v_{1,j}$ for $1 \leq j \leq m_1$ and $w_{1,j}$ for $1 \leq j \leq m_n$ be the adjacent vertices of v_1. Then $|V(G)| = t + 2n + 1$ and $|E(G)| = 2t$.

Define neighbourhood prime labeling $f : V(G) \rightarrow \{1, 2, ..., t + 2n - 1\}$ as follows. Label the vertices $v_1, v_2, ..., v_n$ by odd integers from 1 to $2n - 1$, label the vertices $w_1, w_2, ..., w_{n-1}$ by even integers from 2 to $2n - 2$. Also label the remaining vertices $v_{1,j}(1 \leq j \leq m_1)$, $w_{i,j}(1 \leq j \leq 2n - 2$ and $1 \leq j \leq m_{n-1+i})$,$v_{n,j}(1 \leq j \leq m_n), v_{n-1,j}(1 \leq j \leq m_{n-1}), v_{n-2,j}(1 \leq j \leq m_{n-2}), ..., v_{3,j}(1 \leq j \leq m_2)$ by the remaining integers $r, r + 1, ..., t + 2n + 1$, where $r = m_1 + m_{n-1} + m_{n+1} + ... + m_{3n-3}$. Let x be any vertex of G and $deg(x) > 1$.

Case 1 : $x = v_i$ for $1 \leq i \leq n$.

The set $\{f(z) : z \in N(v_i)$, for $1 \leq i \leq n\}$ contains atleast two consecutive integers. Therefore gcd of the labels of vertices in $N(v_i)$, for $1 \leq i \leq n$ is 1.

Case 2 : $x = w_i$ for $1 \leq i \leq n - 1$

The set $\{f(z) : z \in N(w_i)$, for $1 \leq i \leq n - 1\}$ contains atleast two consecutive integers. Therefore gcd of labels of vertices in $N(w_i)$, for $1 \leq i \leq n - 1$ is 1.

Case 3 : $x = v_{i,j}$ for $1 \leq i \leq n - 1$ and $1 \leq j \leq m_i$

The set $\{f(z) : z \in N(v_{i,j})$, for $1 \leq i \leq n - 1$ and $1 \leq j \leq m_i\}$ contains exactly two consecutive odd integers. Therefore gcd of the labels of vertices in $N(v_{i,j})$, for $1 \leq i \leq n - 1$ and $1 \leq j \leq m_i$ is 1.

Case 4 : $x = w_{i,j}$ for $1 \leq i \leq 2n - 2$ and $1 \leq j \leq m_{n-1+i}$

Since the set $\{f(z) : z \in N(v_{i,j})$, for $1 \leq i \leq 2n - 2$ and $1 \leq j \leq m_{n-1+i}\}$ contains exactly two consecutive integers. Therefore gcd of the labels of vertices in $N(v_{i,j})$, for $1 \leq i \leq 2n - 2$ and $1 \leq j \leq m_{n-1+i}$ is 1.

Hence f is a neighbourhood prime labeling. Thus an arbitrary super subdivision of T_n is neighbourhood prime graph. □

Example 13. An arbitrary super subdivision of triangular snake T_4 and its neighbourhood prime labeling are shown in the Figure 3. Here $m_1 = 2, m_2 = 3, m_3 = 2, m_4 = 2, m_5 = 3, m_6 = 2, m_7 = 2, m_8 = 1$ and $m_9 = 1$.

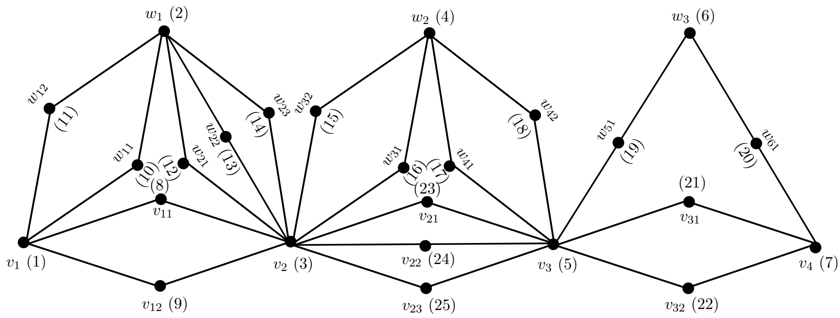

Figure 3

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