International Journal of Pure and Applied Mathematics
Volume 120 No. 4 2018, 657-667
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v120i4.14

ijpam.eu

G-CONE METRIC SPACES AND FIXED POINTS THROUGH MULTIVALUED CONTRACTIONS

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Abstract: The aim of the paper is to obtain a fixed point result for multivalued contraction by using a newly define notion H-cone metric with respect to G for the family $\mathcal{A}$ of subsets of X in complete G-cone metric space. Our result generalizes H-cone metric in the sense of Muhammad Arshad and Jamshaid Ahmad [1] to G-metric spaces. Moreover, an example is provided to illustrate the usability of main result.

AMS Subject Classification: 47H10, 54H25
Key Words: G-cone metric space, H-cone metric, Non-normal cone, Multivalued mapping, Fixed point

1. Introduction

Huang and Zhang [5] preambled the notion of cone metric space by replacing the set of real numbers by a n ordered Banach space using a normal cone with a constant K, which is generalization of a metric space and proved some fixed point theorems of contractive type mappings on cone metric spaces. Thereafter, Klim and Wardowski [7], Latif and Shaddad [9] and Wardowski [16, 17] obtained fixed points of set-valued mappings using normality in cone metric spaces. Rezapour and Hamlbarani [15] presented the results of Huang and Zhang [5] for the case of a cone metric space without using assumption of normality in cone. Jankovic et al. [6] showed that fixed point problems in the

Received: August 12, 2017
Revised: October 10, 2017
Published: December 17, 2018

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setting of cone metric spaces is appropriate only when the underlying cone is non normal, because the results concerning fixed points and other results in the case of cone metric spaces with non normal solid cones cannot be proved by reducing to metric spaces.

To recovered the flaws of Dhage's theory of generalized metric spaces [4], Mustafa and Sims [10] introduced a new structure of generalized metric spaces, which are called G-metric spaces as generalization of metric space (X, d). Afterwards many researchers have studied fixed point results in G-metric spaces [10, 11]. Kaewcharoen and Kaewkhao [8] and Nedal et al. [13] proved fixed point results for multivalued maps in G-metric spaces. In 2010, Beg et al. [2] introduced the notion of G-cone metric space and generalized some results. For the definition of G-cone metric spaces and related concepts we refer the reader to [2, 6, 10].

Let E be a real Banach space and P a subset of E. The subset P is called cone if and only if:

(i) P is closed, non empty and $P \neq \{0\}$;

(ii) $a, b \in R$, $a, b \geq 0$ and $x, y \in P \Rightarrow ax + by \in P$;

(iii) $x \in P$ and $-x \in P \Rightarrow x = 0$.

For a given cone $P \subseteq E$, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in \operatorname{int} P$, where $\operatorname{int} P$ denotes the interior of P. The cone P is said to be solid if it has nonempty interior. The cone P is called normal if there is a number $K > 0$ such that for all $x, y \in E$, $0 \leq x \leq y$ implies $\|x\| = K\|y\|$.

Remark 1.1 ([6]). The results concerning fixed points and other results, in case of cone spaces with non normal solid cones, cannot be provided by reducing to metric spaces, because in this case neither of the conditions of Lemmas 1-4 in [5] hold. Further, the vector cone metric is not continuous in a general case, i.e., from $x_n \to x$, $y_n \to y$ it need not follow that $d(x_n, y_n) \to d(x, y)$.

For the case of non-normal cones, we have the following properties :

(PT1) If $u \leq v$ and $v \ll w$, then $u \ll w$.

(PT2) If $u \ll v$ and $v \leq w$, then $u \ll w$.

(PT3) If $u \ll v$ and $v \ll w$, then $u \ll w$.

(PT4) If $0 \leq u \ll c$ for each $c \in \operatorname{int} P$, then $u = 0$.

(PT5) If $a \leq b + c$ for each $c \in \operatorname{int} P$, then $a \leq b$.

(PT6) If E is a Banach space with a cone P and if $a \leq \lambda a$, where $a \in P$ and $0 \leq \lambda < 1$, then $a = 0$.

(PT7) If $c \in \operatorname{int} P$, $a_n \in E$ and $a_n \to 0$, then there exists an n_0 such that for all $n > n_0$, we have $a_n \ll c$.

Definition 1.2 ([2]). Let X be a nonempty set. Suppose that the mapping $G : X \times X \times X \to E$ satisfies:

(G1) $G(x, y, z) = 0$ if $x = y = z$,

(G2) $0 < G(x, x, y)$, whenever $x \neq y$, for all $x, y \in X$,

(G3) $G(x, x, y) \leq G(x, y, z)$ whenever $y \neq z$,

(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$, (symmetric in all three variables)

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$. (rectangle inequality)

Then the function G is called a generalized cone metric on X and the pair (X, G) is called a generalized cone metric space or more specifically a G-cone metric space.

We use the following proposition in G-cone metric space same as in G-metric space.

Proposition 1.3 ([10]). *Let (X, G) be a G-metric space. Then for any x, y, z and $a \in X$ it follows that:*

(1) *If $G(x, y, z) = 0$, then $x = y = z$,*

(2) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$,

(3) $G(x, y, y) \leq 2G(y, x, x)$,

(4) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$,

(5) $G(x, y, z) \leq \frac{2}{3}(G(x, y, a) + G(x, a, z) + G(a, y, z))$,

(6) $G(x, y, z) \leq G(x, a, a) + G(y, a, a) + G(z, a, a))$.

Definition 1.4 ([2]). A G-cone metric space X is symmetric if $G(x, y, y) = G(y, x, x)$ for all $x, y \in X$.

Example 1.5 ([2]). Let (X, d) be a cone metric space. Define $G : X \times X \times X \to E$ by $G(x, y, z) = d(x, y) + d(y, z) + d(z, x)$. Then (X, G) is a G-cone metric space.

Proposition 1.6 ([2]). *Let X be a G-cone metric space, define $d_G : X \times X \to E$ by*

$$d_G(x, y) = G(x, y, y) + G(y, x, x).$$

Then (X, d_G) is a cone metric space. It can be noted that $G(x, y, y) \le \frac{2}{3} d_G(x, y)$. If X is a symmetric G-cone metric space, then $d_G(x, y) = 2G(x, y, y)$ for all $x, y \in X$.

Definition 1.7 ([2]). Let X be a G-cone metric space and let $\{x_n\}$ be a sequence in X, then $\{x_n\}$ is:

(i) A Cauchy sequence if for every $c \in E$ with $0 \ll c$ there is N such that for all $n, m, l > N$, $G(x_n, x_m, x_l) \ll c$.

(ii) A convergent sequence if for every $c \in E$ with $0 \ll c$ there is N such that for all $m, n > N$, $G(x_m, x_n, x) \ll c$ for some fixed x in X. Here x is called the limit of a sequence $\{x_n\}$ and is denoted by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

A G-cone metric space X is said to be complete if every Cauchy sequence in X is convergent in X.

Proposition 1.8 ([2]). *Let X be a G-cone metric space then the following are equivalent.*

(i) *$\{x_n\}$ is converges to x,*

(ii) *$G(x_n, x_n, x) \to 0$ as $n \to \infty$,*

(iii) *$G(x_n, x, x) \to 0$ as $n \to \infty$,*

(iv) *$G(x_m, x_n, x) \to 0$ as $m, n \to \infty$.*

Lemma 1.9 ([2]). *Let $\{x_n\}$ be a sequence in a G-cone metric space X. If $\{x_n\}$ converges to $x \in X$, then $G(x_m, x_n, x) \to 0$, as $m, n \to \infty$.*

Lemma 1.10 ([2]). *Let $\{x_n\}$ be a sequence in a G-cone metric space X and $x \in X$. If $\{x_n\}$ converges to $x \in X$, then $\{x_n\}$ is a Cauchy sequence.*

Lemma 1.11 ([2]). *Let $\{x_n\}$ be a sequence in a G-cone metric space X. If $\{x_n\}$ is a Cauchy sequence in X, then $G(x_m, x_n, x_l) \to 0$, as $m, n, l \to \infty$.*

Remark 1.12 ([8]). Kaewcharoen and Kaewkhao [8] established the following concepts:

Let X be a G-metric space and let $CB(X)$ be the family of all nonempty closed bounded subsets of X. Let $H_G(\cdot,\cdot,\cdot)$ be the Hausdorff G-distance on $CB(X)$, i.e.,

$$H_G(A,B,C) = \max\left\{\sup_{a\in A} G(a,B,C), \sup_{b\in B} G(b,A,C), \sup_{c\in C} G(c,A,B)\right\},$$
$$H_{d_G}(A,B) = \max\left\{\sup_{a\in A} d_G(a,B), \sup_{b\in B} d_G(b,A)\right\},$$

where

$$\begin{aligned} &G(x,B,C) = d_G(x,B) + d_G(B,C) + d_G(x,C),\\ &d_G(x,B) = \inf\{d_G(x,y), y\in B\},\\ &d_G(A,B) = \inf\{d_G(a,b), a\in A, b\in B\},\\ &G(a,b,C) = \inf\{G(a,b,c), c\in C\}.\end{aligned}$$

The above expressions show a relation between H_G and H_{d_G}. Moreover, note that if (X,G) is a G-cone metric space, $E=R$, and $P=[0,\infty)$, then (X,G) is a G-metric space.

Let X and Y be non empty sets. T is said to be a multivalued mapping from X to Y if T is a function from X to the power set of Y. We denote a multivalued mapping by $X \to 2^y$. A point $x \in X$ is said to be a fixed point of multivalued mapping T if $x \in Tx$. We denote the set of fixed points of T by Fix(T).

The Nadler's result [12] concerning set-valued contractive mappings in metric spaces become the inspiration for many authors in the metric fixed point theory. In 2011, Wardowski [16] introduced a new cone metric $H : \mathcal{A}\times\mathcal{A} \to E$ for a normal cone metric space (X,d) and proved a fixed point theorem for the family $\mathcal{A}$ of subset of X by introducing the concept of set valued contraction of Nadler type. Arshad and Ahmad [1] minutely convert the idea of H-cone metric to make it more comparable with a standard metric and its result generalized in case of cone b-metric by [14]. Chi-Ming Cheng [3] proved Nadler type results in tvs G-cone metric spaces.

In this paper, for a G-cone metric space (X,G) and for the family $\mathcal{A}$ of subsets of X, we introduce a notion of H-cone metric H with respect to G and establish a fixed point result for multivalued mapping in a complete G-cone metric space. Our result generalizes some recent results due to Wardowski [16] and Arshad and Ahmad [1].

2. Main Result

Definition 2.1. Let (X, G) be a G-cone metric space with the cone P. A mapping $H : \mathcal{A} \times \mathcal{A} \times \mathcal{A} \to E$ where be a collection of nonempty subset of X is called an H-cone metric with respect to G if for any $A, B, C \in \mathcal{A}$ the following condition hold:

(H1) $H(A, B, C) = 0$ if $A = B = C$,

(H2) $0 < H(A, A, B)$, whenever $A \neq B$, for all $A, B \in \mathcal{A}$,

(H3) $H(A, A, B) \leq H(A, B, C)$, whenever $B \neq C$,

(H4) $H(A, B, C) = H(A, C, B) = H(B, A, C) = \ldots$ (symmetric in all three variables),

(H5) $H(A, B, C) \leq H(A, a, a) + H(a, B, C)$ for all $A, B, C, a \in \mathcal{A}$,

(H6) If $A, B, C \in \mathcal{A}$, $0 < \varepsilon \in E$ with $H(A, B, B) < \varepsilon$, then for each $a \in A$ there exist $b \in B$ such that $G(a, b, b) < \varepsilon$.

Example 2.2. Let (X, G) be a G-cone metric space and $\mathcal{A} = \{\{x\} : x \in X\}$. Then the mapping given by the formula $H(\{x\}, \{y\}, \{z\}) = G(x, y, z)$ is an H-cone metric with respect to G.

Example 2.3. Let (X, G) be a G-cone metric space and $\mathcal{A}$ be a family of all nonempty closed bounded subsets of X. Then $H : \mathcal{A} \times \mathcal{A} \times \mathcal{A} \to E$ given by the formula

$$H_G(A, B, C) = \max \left\{ \sup_{a \in A} G(a, B, C), \sup_{b \in B} G(b, A, C), \sup_{c \in C} G(c, A, B) \right\},$$

where $A, B, C \in \mathcal{A}$ is a H-cone metric (Hausdorff-Pompeiu metric) with respect to G.

Theorem 2.4. *Let (X, G) be a complete G-cone metric space. Let $\mathcal{A}$ be a collection of nonempty closed subsets of X and $H : \mathcal{A} \times \mathcal{A} \times \mathcal{A} \to E$ be a H-cone metric induced by G. If for mapping $T : X \to \mathcal{A}$ there exist $\lambda \in (0, 1)$ such that for all $x, y, z \in X$,*

$$H(Tx, Ty, Tz) \leq \lambda G(x, y, z) \tag{2.1}$$

then $\operatorname{Fix} T \neq 0$.

Proof. Let x_0 be an arbitrary but fixed element of X and $x_1 \in Tx_0$. If $x_0 = x_1$, then $x_0 \in \operatorname{Fix} T$ and if $x_0 \neq x_1$, using the fact that

$$H(Tx_0, Tx_1, Tx_1) \leq \lambda G(x_0, x_1, x_1) < \sqrt{\lambda} G(x_0, x_1, x_1), \tag{2.2}$$

we may choose $x_2 \in X$ such that $x_2 \in Tx_1$ and

$$G(x_1, x_2, x_2) < \sqrt{\lambda} G(x_0, x_1, x_1). \tag{2.3}$$

Similarly, in case $x_1 \neq x_2$ we may choose $x_3 \in X$ such that $x_3 \in Tx_2$ and

$$G(x_2, x_3, x_3) < \sqrt{\lambda} G(x_1, x_2, x_2) < (\sqrt{\lambda})^2 G(x_0, x_1, x_1). \tag{2.4}$$

We can continue this process to find a sequence $\{x_n\}$ of points of X such that

$$\begin{aligned} x_{n+1} &\in Tx_n \quad \text{for } n = 0, 1, 2, \dots \\ G(x_n, x_{n+1}, x_{n+1}) &< \sqrt{\lambda} G(x_{n-1}, x_n, x_n) \\ &< (\sqrt{\lambda})^2 G(x_{n-2}, x_{n-1}, x_{n-1}) \\ &\dots \\ &< (\sqrt{\lambda})^n G(x_0, x_1, x_1). \end{aligned} \tag{2.5}$$

Now for any $m > n$,

$$\begin{aligned} G(x_n, x_m, x_m) &\leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_m, x_m) \\ &\leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) \\ &\quad + G(x_{n+2}, x_m, x_m) \\ &\leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) \\ &\quad + G(x_{n+2}, x_{n+3}, x_{n+3}) + \dots + G(x_{m-1}, x_m, x_m) \\ &\leq (\sqrt{\lambda})^n G(x_0, x_1, x_1) + (\sqrt{\lambda})^{n+1} G(x_0, x_1, x_1) + \dots \\ &\quad + (\sqrt{\lambda})^{m-1} G(x_0, x_1, x_1) \\ &\leq [(\sqrt{\lambda})^n + (\sqrt{\lambda})^{n+1} + \dots + (\sqrt{\lambda})^{m-1}] G(x_0, x_1, x_1) \\ &\leq \left[\frac{(\sqrt{\lambda})^n}{1 - \sqrt{\lambda}}\right] G(x_0, x_1, x_1). \end{aligned} \tag{2.6}$$

Let $0 \ll c$ be given. Choose a symmetric neighborhood V of 0 such that $c + V \subseteq \operatorname{int} P$. Also, choose a natural number N_1 such that $\left[\frac{(\sqrt{\lambda})^n}{1-\sqrt{\lambda}}\right] G(x_0, x_1, x_1) \in V$, for all $n \geq N_1$. Then $\left[\frac{(\sqrt{\lambda})^n}{1-\sqrt{\lambda}}\right] G(x_0, x_1, x_1) \ll c$, for all $n \geq N_1$. Thus,

$$G(x_n, x_m, x_m) \leq \left[\frac{(\sqrt{\lambda})^n}{1-\sqrt{\lambda}}\right] G(x_0, x_1, x_1) \ll c \tag{2.7}$$

for all $m > n$. Therefore, $\{x_n\}_{n\geq 1}$ is a Cauchy sequence. Since (X, G) is a G-cone complete metric space. There exists $u \in X$ such that $x_n \to u$. Since

$$H(Tx_n, Tx_n, Tu) \leq \lambda G(x_n, x_n, u) < \sqrt{\lambda} G(x_n, x_n, u) \tag{2.8}$$

for each n, $x_{n+1} \in Tx_n$, we have $y_n \in Tu$ such that
$G(x_{n+1}, x_{n+1}, y_n) < \sqrt{\lambda} G(x_n, x_n, u)$.

Now, choose a natural number N_2 such that

$$G(x_n, x_n, u) \ll \frac{c}{2} \quad \text{for all } n \geq N_2 \tag{2.9}$$

Then for all $n \geq N_2$,

$$\begin{aligned} G(y_n, u, u,) &\leq G(y_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, u, u) \\ &\leq G(y_n, x_{n+1}, x_{n+1}) + \sqrt{\lambda} G(x_n, x_n, u) \\ &\leq G(y_n, x_{n+1}, x_{n+1}) + G(x_n, x_n, u) \\ &\ll \frac{c}{2} + \frac{c}{2} = c. \end{aligned} \tag{2.10}$$

It follows that $y_n \to u$, and it implies that $u \in Tu$. □

Example 2.5. Suppose $X = [0, 1]$, $E = C^2_R[0, 1]$ with the norm $\|x\| = \|x\|_\infty + \|x'\|_\infty$, $P = \{x \in E : x \geq 0\}$, $x(t) = t$ and $y(t) = t^{2K}$. Then $0 \leq x \leq y$, $\|x\| = 2$ and $\|y\| = 1 + 2K$. For all $K \geq 1$, since $K\|x\| < \|y\|$. Therefore, P is non normal cone. Define $G : X \times X \times X \to E$ as follows:

$$G(x, y, z)(t) = \max\{|x - y|, |y - z|, |x - z|\}e^t$$

Then G is a G-cone metric on X. Let $\mathcal{A}$ be a family of subsets of X of the form $\mathcal{A} = \{[0, x] : x \in X\}$ and define $H : \mathcal{A} \times \mathcal{A} \times \mathcal{A} \to E$ as follow

$$H(A, B, C) = \max\{2(|x - y| + |x - z|, |x - y| + |y - z|, |x - z| + |y - z|)\}e^t$$

where $A, B, C \in \mathcal{A}$ and $x, y, z \in X$. It is easy to observe that H satisfy (H1)-(H6) of Definition 2.1

Define $T : X \to \mathcal{A}$ as $Tx = \left[0, \frac{x}{12}\right]$, by using

$$d_G(x,y) = G(x,y,y) + G(y,x,x),$$

we have

$$d_G(x,y) = 2|x-y|e^t \quad \text{for all } x,y \in X.$$

To prove (2.1), let $x,y,z \in X$. If $x = y = z = 0$, then

$$H(Tx,Ty,Tz) = 0 \leq \lambda G(x,y,z).$$

Thus we assume that x, y and z are not all zero. Without loss of generality, we assume that $x \leq y \leq z$. Then

$$\begin{aligned} H(Tx,Ty,Tz) &= H\left(\left[0,\frac{x}{12}\right],\left[0,\frac{y}{12}\right],\left[0,\frac{z}{12}\right]\right) \\ &= \max\left\{\sup_{0\leq a\leq\frac{x}{12}} G\left(a,\left[0,\frac{y}{12}\right],\left[0,\frac{z}{12}\right]\right),\right. \\ &\qquad \sup_{0\leq b\leq\frac{y}{12}} G\left(b,\left[0,\frac{z}{12}\right],\left[0,\frac{x}{12}\right]\right), \\ &\qquad \left.\sup_{0\leq c\leq\frac{z}{12}} G\left(c,\left[0,\frac{x}{12}\right],\left[0,\frac{y}{12}\right]\right)\right\}. \end{aligned}$$

Since $x \leq y \leq z$, so $\left[0, \frac{x}{12}\right] \subseteq \left[0, \frac{y}{12}\right] \subseteq \left[0, \frac{z}{12}\right]$.

This implies that

$$\begin{aligned} d_G\left(\left[0,\frac{x}{12}\right],\left[0,\frac{y}{12}\right]\right) &= d_G\left(\left[0,\frac{y}{12}\right],\left[0,\frac{z}{12}\right]\right) \\ &= d_G\left(\left[0,\frac{x}{12}\right],\left[0,\frac{z}{12}\right]\right) \\ &= 0. \end{aligned}$$

For each $0 \leq a \leq \frac{x}{12}$, we have

$$\begin{aligned} &G\left(a,\left[0,\frac{y}{12}\right],\left[0,\frac{z}{12}\right]\right) \\ &= d_G\left(a,\left[0,\frac{y}{12}\right]\right) + d_G\left(\left[0,\frac{y}{12}\right],\left[0,\frac{z}{12}\right]\right) + d_G\left(a,\left[0,\frac{z}{12}\right]\right) \\ &= 0. \end{aligned}$$

Also for each $0 \leq b \leq \frac{y}{12}$, we have

$$G\left(b,\left[0,\frac{z}{12}\right],\left[0,\frac{x}{12}\right]\right)$$

$$= d_G\left(b, \left[0, \frac{z}{12}\right]\right) + d_G\left(\left[0, \frac{z}{12}\right], \left[0, \frac{x}{12}\right]\right) + d_G\left(b, \left[0, \frac{x}{12}\right]\right)$$

$$= \begin{cases} 0 & \text{if } b \leq \frac{x}{12} \\ 2\left|b - \frac{x}{12}\right| e^t & \text{if } b \geq \frac{x}{12} \end{cases}$$

This yields that

$$\sup_{0 \leq b \leq \frac{y}{12}} G\left(b, \left[0, \frac{z}{12}\right], \left[0, \frac{x}{12}\right]\right) = 2\left|\frac{y}{12} - \frac{x}{12}\right| e^t.$$

Moreover, for each $0 \leq c \leq \frac{z}{12}$, we have

$$G\left(c, \left[0, \frac{x}{12}\right], \left[0, \frac{y}{12}\right]\right)$$

$$= d_G\left(c, \left[0, \frac{x}{12}\right]\right) + d_G\left(\left[0, \frac{x}{12}\right], \left[0, \frac{y}{12}\right]\right) + d_G\left(c, \left[0, \frac{y}{12}\right]\right)$$

$$= \begin{cases} 0 & \text{if } c \leq \frac{x}{12} \\ 2\left|c - \frac{x}{12}\right| e^t & \text{if } \frac{x}{12} \leq c \leq \frac{y}{12} \\ 2\left|2c - \frac{x}{12} - \frac{y}{12}\right| e^t & \text{if } c \geq \frac{y}{12} \end{cases}$$

This yield that

$$\sup_{0 \leq c \leq \frac{z}{12}} G\left(c, \left[0, \frac{x}{12}\right], \left[0, \frac{y}{12}\right]\right) = 2\left|\frac{z}{6} - \frac{x}{12} - \frac{y}{12}\right| e^t.$$

We deduce that

$$\begin{aligned} H(Tx, Ty, Tz) &= 2\left|\frac{z}{6} - \frac{x}{12} - \frac{y}{12}\right| e^t \quad \text{for } t \in [0,1] \\ &\leq \frac{1}{3}|z - x| e^t \\ &\leq \frac{1}{3} Max\{|x-y|, |y-z|, |x-z|\} e^t \\ &= \frac{1}{3} G(x, y, z). \end{aligned}$$

Note that T satisfies the conditions of Theorem with $\lambda = \frac{1}{3}$ and $0 \in \operatorname{Fix} T$.

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