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MIXED γ-FUZZY IN MIXED FUZZY TOPOLOGICAL SPACES AND ITS APPLICATIONS

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Abstract: We deal with fuzzy topology. In this paper, we introduce and study the notion of mixed γ-fuzzy on mixed fuzzy topological spaces. We have investigated this notion in the light of the notion of γ-q-neighbourhoods, γ-q-coincidence, fuzzy γ-closure, fuzzy γ-interior. Further, some relations are established between the two topologies used and their corresponding mixed fuzzy topological spaces.

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Key Words: Mixed topology, fuzzy γ-open set, fuzzy γ-closed set, fuzzy γ-q-nbd, fuzzy γ-closure

1. Introduction

1965 L.A. Zadeh introduced the concept of fuzzy sets. Since then the notion of fuzziness has been applied for the study in all the branches of science and technology. It has been applied in mathematical analysis for introducing and investigating different classes of sequence spaces of fuzzy numbers by Tripathy and Baruah [5, 6], and many others in the recent past. The notion of fuzziness has been applied in topology and the notion of fuzzy topological spaces has introduced and investigated by many researches on topological spaces. Different properties of fuzzy topological spaces have been investigated by Arya and Singal [1, 2], Chang [3], Das and Baishya [4],Tripathy and Debnath [7] and many

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others. Recently mixed fuzzy topological spaces has been investigated from different aspect by Das and Baishya [4], Tripathy and Ray [8, 9] and others.

The study of mixed topology originated from the work of Polish Mathematicians Alexiewicz and Semadini. The study of mixed topology was first invented by Finchtenholz of the Polish school of Mathematics in the year 1938. Mixed topology is a technique of mixing two topologies on the same set in order to obtain a third topology. N. R. Das and P. C. Baishya [10] have constructed a fuzzy topology called mixed fuzzy topology, from the two given topologies on a set $\mathcal{X}$ with the help of closure of neighborhoods of one topology with respect to the other topology and then studied various properties of this topology. Azad [13] has defined fuzzy defined fuzzy regular open (fuzzy regular closed). Benchalli and Jenifer [12] defined the concepts of fuzzy γ-open set and fuzzy γ-continuous mappings in fuzzy topological spaces. This paper deals with the construction of a topological space. In this paper the work of N. R. Das and P. C. Baishya [10] is used to introduce and study the newly constructed mixed γ-fuzzy topological space. This topological space is constructed from two different fuzzy topologies using fuzzy γ-q-nbd of a fuzzy point with respect to one topology and fuzzy γ-closure of a fuzzy set with respect to the another topology. Then it is proved to satisfy the four conditions of a fuzzy topological space. Under certain conditions, some relations are established between the two topologies used and their corresponding mixed topology.

Most of the concepts, notations and definitions which we have used in this paper are standard by now. But, for the sake of completeness we recall some definitions and results used in the sequel.

2. Preliminaries and Definitions

Definition 1. [3] A fuzzy set in $\mathcal{X}$ is called a fuzzy point if and only if it takes the value 0 for all $y \in \mathcal{X}$ except one, say $x \in \mathcal{X}$. If its value at x is λ $(0 < \lambda < 1)$, we denote this fuzzy point by x_λ, where the point x is called its support.

Definition 2. [3] A fuzzy set λ in a fuzzy topological space $\mathcal{X}$ is said to be quasi-coincident (q-coincident, in short) with a fuzzy set μ in $\mathcal{X}$, denoted by $\lambda q \mu$, if there exists some $x \in \mathcal{X}$ such that $\lambda(x) + \mu(x) > 1$. If λ is not q-coincident with μ, we write $\lambda \widetilde{q} \mu$.

If $(\mathcal{X}, \mathcal{T})$ is a fuzzy topological space, then the closure and interior of a fuzzy set λ in $\mathcal{X}$, denoted by $Cl(\lambda)$ and $Int(\lambda)$ respectively, are defined by $Cl(\lambda) = \cap\{\mu : \mu \text{is a closed fuzzy set in} \, \mathcal{X} \, \text{and} \, A \subseteq \mu\}$ and $Int(\lambda) = \cap\{\nu :$

$\nu$is an open fuzzy set in $\mathcal{X}$
and $\nu \subseteq \lambda\}$. Clearly, $Cl(\lambda)$ (respectively $Int(\lambda)$) is the smallest (respectively largest) closed(respectively open) fuzzy set in X containing (respectively contained in) λ. If there are more than one topologies on X, then the closure and interior of λ with respect to a fuzzy topology τ on X will be denoted by $\tau - Cl(\lambda)$ and $\tau - Int(\lambda)$. A fuzzy subset λ of a space X is called fuzzy-preopen [14] if $\lambda \leq Int(Cl(\lambda))$. The complement of a fuzzy-preopen set is called fuzzy-pre-closed.

Definition 3. A fuzzy point x_p is said to be quasi-coincident with λ, denoted by $x_p q \lambda$, if and only if $\alpha + \lambda(x) > 1$ or $\alpha > (\lambda(x))^c$.

It is clear from the above definition, if A and B are quasicoincident at x both $\lambda(x)$ and $\mu(x)$ are not zero at x and hence λ and μ intersect at x.

Definition 4. A fuzzy set λ in a fuzzy topological space $\mathcal{X}$ is called fuzzy γ-open [11] if $\lambda \leq Int(Cl(\lambda)) \vee Cl(Int(\lambda))$. The complement of fuzzy γ-open set is called fuzzy γ-closed. (i. e. $Int(Cl(\lambda)) \wedge Cl(Int(\lambda)) \geq \lambda$).

Definition 5. A fuzzy set λ in a fts $(\mathcal{X}, \mathcal{T})$ is called a quasi-neighborhood of x_λ if and only if $\lambda_1 \in \mathcal{T}$ such that $\lambda_1 \subseteq \lambda$ and $x_\alpha q A_1$. The family of all Q-neighborhoods of x_λ is called the system of Q-neighborhood of x_λ. Intersection of two quasi-neighborhoods of x_λ is a quasi-neighborhood of x_λ. Let $(\mathcal{X}, \mathcal{T}_1)$ and $(\mathcal{X}, \mathcal{T}_2)$ be two fuzzy topological spaces and let $\mathcal{T}_1(\mathcal{T}_2)$ be defined as follows.
$\mathcal{T}_1(\mathcal{T}_2) = \{\lambda \in I^{\mathcal{X}}$: for every fuzzy set μin $\mathcal{X}$ with $\lambda q \mu$, there exists a $\mathcal{T}_2$-open set λ_α, such that $\lambda_\alpha q \mu$ and $\mathcal{T}_1$-closure, $\lambda_\alpha \subseteq \mu\}$.
Then $\mathcal{T}_1(\mathcal{T}_2)$ is a topology on $\mathcal{X}$ and this is called mixed fuzzy topology and the space $(\mathcal{X}, \mathcal{T}_1(\mathcal{T}_2))$ is called mixed fuzzy topological space is introduced and studied by Tripathy and Ray [8].

Lemma 6. *In a fuzzy topological space we have the following*
(a) Every fuzzy regular open set is fuzzy open [13].
(b) Every fuzzy pre-open set is fuzzy γ-open [11].

Definition 7. [11, 13] Let λ be a fuzzy set in a fts $\mathcal{X}$ then,
$\gamma Cl(\lambda) = \wedge\{\mu \geq \lambda$: μ is a $f\gamma$-closed set of $\mathcal{X}\}$.
$\gamma Int(\lambda) = \vee\{\nu \leq \lambda$: ν is a $f\gamma$-open set of $\mathcal{X}\}$.

Theorem 8. *[11] In a fts $\mathcal{X}$ λ is $f\gamma$-open ($f\gamma$-closed) if and only if $\lambda = f\gamma Cl(\lambda)$ ($f\gamma Int(\lambda)$).*

3. Mixed γ-fuzzy topological spaces

Definition 9. [12] Let λ be a fuzzy set in a fts $\mathcal{X}$ and x_p be a fuzzy point of $\mathcal{X}$. Then λ is called
(a) γ-neighbourhood of x_p if there exists a $f\gamma$-open set μ in $\mathcal{X}$ such that $x_p \in \mu \leq \lambda$.
(b) γ-q-neighbourhood of x_p if there exist a $f\gamma$-open set μ in $\mathcal{X}$ such that $x_p q\mu \leq \lambda$.

Definition 10. Let $(\mathcal{X}, \mathcal{T}_1)$ and $(\mathcal{X}, \mathcal{T}_2)$ be two fuzzy topological spaces satisfying the condition that intersection of two fuzzy γ-open sets is fuzzy γ-open and let $\mathcal{T}_1(\mathcal{T}_2) = \{\lambda \in I^{\mathcal{X}} \mid$ for every $x_p q\lambda$, there exists a fuzzy $\mathcal{T}_2$-γ-q-nbd λ_α of x_p such that $\mathcal{T}_1$-$\gamma Cl(\lambda_\alpha) \subseteq \lambda\}$. Then $\mathcal{T}_1(\mathcal{T}_2)$ is a fuzzy topology on $\mathcal{X}$ called mixed γ-fuzzy topology and $(\mathcal{X}, \mathcal{T}_1(\mathcal{T}_2))$ is called mixed γ-fuzzy topological space.

Lemma 11. *Intersection of two fuzzy γ-q-nbds of a fuzzy point x_p is again a fuzzy γ-q-nbd of x_p in a fuzzy topological spaces satisfying the condition that intersection of two fuzzy γ-open sets is fuzzy γ-open.*

Proof. Let $(\mathcal{X}, \mathcal{T}_1)$ be a fuzzy topological space satisfying the condition that intersection of two fuzzy γ-open sets is fuzzy γ-open. Let A and B be two fuzzy γ-q-nbd of a fuzzy point x_p. So, there exists fuzzy γ-open sets μ_1 and μ_2 such that $x_p q\mu_1 \leq \lambda$ and $x_p q\mu_2 \leq \lambda$. Therefore, $\alpha + \mu_1(x) > 1$ and $\alpha + \mu_2(x) > 1$ which implies $\alpha + min\{\mu_1(x), \mu_2(x)\} > 1$. So, $\alpha + (\mu_1 \backslash \mu_2)(x) > 1$. So,

$$x_p q(\mu_1 \cap \mu_2) \qquad (1)$$

Again since $\mu_1 \leq \lambda$ and $\mu_2 \leq \mu$, therefore

$$\mu_1 \cap \mu_2 \leq \lambda \cap \mu \qquad (2)$$

Also $\mu_1 \cap \mu_2$ is fuzzy γ-open since intersection of two fuzzy γ-open sets is fuzzy γ-open by our assumption. Thus, combining this result with equations (1) and (2) we conclude that $\mu_1 \cap \mu_2$ is a fuzzy γ-qnbd of x_p. This completes the proof. □

Theorem 12. *Let $(\mathcal{X}, \mathcal{T}_1)$ and $(\mathcal{X}, \mathcal{T}_2)$ be two fuzzy topological spaces satisfying the condition that intersection of two fuzzy γ-open sets is fuzzy γ-open and let $\mathcal{T}_1(\mathcal{T}_2) = \{A \in I^{\mathcal{X}} \mid$ for every $x_p q\lambda$, there exists a fuzzy $\mathcal{T}_2$-γ-q-nbd λ_α of x_p such that $\mathcal{T}_1$-$\gamma Cl(\lambda_\alpha) \subseteq A\}$. Then $\mathcal{T}_1(\mathcal{T}_2)$ is a fuzzy topology on $\mathcal{X}$.*

Proof. (i) To show $0_{\mathcal{X}}$, $1_{\mathcal{X}} \in \mathcal{T}_1(\mathcal{T}_2)$. No fuzzy point is q-coincident with $0_{\mathcal{X}}$ since $\alpha + o_{\mathcal{X}}1$. As no fuzzy point q-coincident with $0_{\mathcal{X}}$, violates the condition of being a member of $\mathcal{T}_1(\mathcal{T}_2)$, we conclude that $0_{\mathcal{X}} \in \mathcal{T}_1(\mathcal{T}_2)$. Again we know that $\alpha + 1_{\mathcal{X}} > 1$. Therefore $x_p q 1_{\mathcal{X}} \leq 1_{\mathcal{X}}$.
Let $\lambda_\alpha = 1_{\mathcal{X}}$. Then $\gamma Cl(\lambda_\alpha) = \gamma Cl(1_{\mathcal{X}}) = 1_{\mathcal{X}}(1_{\mathcal{X}} \leq \gamma Cl(1_{\mathcal{X}})$ using Remark 7 but $1_{\mathcal{X}}$ being the largest set $1_{\mathcal{X}} = \gamma Cl(1_{\mathcal{X}})$. Thus $1_{\mathcal{X}} \in \mathcal{T}_1(\mathcal{T}_2)$.

(ii) To show $\bigcup_\xi \lambda_\xi \in \mathcal{T}_1(\mathcal{T}_2)$ where $\{\lambda_\xi\}$ is a collection of members of $\mathcal{T}_1(\mathcal{T}_2)$. Let $\{\lambda_\xi\}$ is a collection of members of $\mathcal{T}_1(\mathcal{T}_2)$. To show for every $x_p q \bigcup_\xi \lambda_\xi$, there exists fuzzy $\mathcal{T}_2$-γ-q-nbd λ_α of x_p such that $\mathcal{T}_1$-$\gamma Cl(\lambda_\alpha) \leq \bigcup_\xi \lambda_\xi$. Suppose we consider a fuzzy point x_p. Now, $x_p q \bigcup_\xi \lambda_\xi$ implies $\alpha + \bigcup_\xi \lambda_\xi > 1$. This implies $\alpha + \text{Sup}_\xi \lambda_\xi > 1$ which in turn implies $\text{Sup}_\xi \lambda_\xi > 1 - \alpha$. Therefore, there exists ξ_0 such that $\alpha + \lambda_{\xi_0}(x) > 1$. This implies $\lambda_{\xi_0}(x) > 1 - \alpha$. That is $x_p q \lambda_{\xi_0}$. We know $\lambda_{\xi_0} \in \mathcal{T}_1(\mathcal{T}_2)$. Therefore, there exists, a fuzzy $\mathcal{T}_2$-γ-q-nbd λ_{ζ_0} of x_p such that $\mathcal{T}_1$-$\gamma Cl(\lambda_{\zeta_0}) \leq \lambda_{\xi_0} \leq \bigcup_\xi \lambda_\xi$. Therefore $\bigcup_\xi \lambda_\xi \in \mathcal{T}_1(\mathcal{T}_2)$.

(iii) To prove $(\lambda_1 \cap \lambda_2) \in \mathcal{T}_1(\mathcal{T}_2)$, where λ_1, $\lambda_2 \in \mathcal{T}_1(\mathcal{T}_2)$, that is to prove for every $x_p q(\lambda_1 \cap \lambda_2)$, there exists a fuzzy $\mathcal{T}_2$-γ-q-nbd λ_α of x_p such that $\mathcal{T}_1$-$\gamma Cl(\lambda_\alpha) \leq (\lambda_1 \cap \lambda_2)$. Let $x_p q(\lambda_1 \cap \lambda_2)$. This implies $\alpha + (\lambda_1 \cap \lambda_2)(x) > 1$ which implies $\alpha + \min\{\lambda_1(x), \lambda_2(x)\} > 1$. So $\alpha + \lambda_1(x) > 1$ and $\alpha + \lambda_2(x) > 1$. Therefore, $x_p q \lambda_1$ and $x_p q \lambda_2$. Now $\lambda_1 \in \mathcal{T}_1(\mathcal{T}_2)$. This implies that there exists a fuzzy $\mathcal{T}_2$-γ-q-nbd λ_{α_1} of x_p such that

$$\mathcal{T}_1 - \gamma Cl(\lambda_{\alpha_1}) \leq (\lambda_1. \qquad (3)$$

Also $\lambda_2 \in \mathcal{T}_1(\mathcal{T}_2)$ which implies there exists a fuzzy $\mathcal{T}_2$-γ-q-nbd λ_{α_2} of x_p such that

$$\mathcal{T}_1 - \gamma Cl(\lambda_{\alpha_2}) \leq (\lambda_2. \qquad (4)$$

Since λ_{α_1} and λ_{α_2} are fuzzy $\mathcal{T}_2$-γ-q-nbd of x_p, therefore $\lambda_{\alpha_1} \cap \lambda_{\alpha_2}$ is a fuzzy $\mathcal{T}_2$-γ-q-nbd of x_p, using Lemma 11. Now (3) and (4) implies

$$(\mathcal{T}_1 - \gamma Cl(\lambda_{\alpha_1})) \cap (\mathcal{T}_2 - \gamma Cl(\lambda_{\alpha_2})) \leq (\lambda_1 \cap \lambda_2) \qquad (5).$$

Finally, to prove $\mathcal{T}_1 - \gamma Cl(\lambda_{\alpha_1} \cap \lambda_{\alpha_2}) \leq (\mathcal{T}_1 - \gamma Cl(\lambda_{\alpha_1})) \cap (\mathcal{T}_2 - \gamma Cl(\lambda_{\alpha_2}))$. Let $x_\delta \in \gamma Cl(\lambda_{\alpha_1} \cap \lambda_{\alpha_2})$. This implies that $\lambda_{\alpha_1} \cap \lambda_{\alpha_2}$ is a γ-neighbourhood of x_δ, which implies that every fuzzy γ-q-nbd U of x_δ is q-coincident with $(\lambda_{\alpha_1} \cap \lambda_{\alpha_2})$, therefore, $Uq(\lambda_{\alpha_1} \cap \lambda_{\alpha_2})$ which implies $U(x) + (\lambda_{\alpha_1} \cap \lambda_{\alpha_2})(x) > 1$, that is $U(x) + \min(\lambda_{\alpha_1}(x) \cap \lambda_{\alpha_2}(x)) > 1$. Thus $U(x) + \lambda_{\alpha_1}(x) > 1$ and $U(x) + \lambda_{\alpha_2}(x) > 1$. This implies that every fuzzy γ-q-nbd U of x_δ is q-coincident with λ_{δ_1} and λ_{δ_2}. Thus x_δ is a fuzzy point of λ_{α_1} and that of λ_{α_2}. Therefore, $x_\delta \in \gamma Cl(\lambda_{\alpha_1})$ and $x_\delta \in \gamma Cl(\lambda_{\alpha_2})$. So $x_\delta \in \gamma Cl(\lambda_{\alpha_1}) \cap \gamma Cl(\lambda_{\alpha_2})$. Therefore, $\gamma Cl(\lambda_{\alpha_1} \cap \lambda_{\alpha_2}) \leq \gamma Cl(\lambda_{\alpha_1}) \cap \gamma Cl(\lambda_{\alpha_2}) \leq \lambda_1 \cap \lambda_2$ using (5). Therefore, $\lambda_1 \cap \lambda_2 \in \mathcal{T}_1(\mathcal{T}_2)$, where

$\lambda_1,\ \lambda_2 \in \mathcal{T}_1(\mathcal{T}_2)$. Therefore, $\mathcal{T}_1(\mathcal{T}_2)$ is a fuzzy topology on $\mathcal{X}$. This proves the Theorem. □

We now characterize a Coarser topology in the light of γ-q-neighbourhoods

4. Conclusion

We have introduced Mixed γ-fuzzy Topological Spaces and Fuzzy Completely Weakly γ-Irresolute Function over an initial universe with a fixed set of parameters. Many results have been established to show how far topological structures are preserved by these γ-Irresolute Functions. We also have provided examples where such properties fail to be preserved. In this paper, we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application.

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