International Journal of Pure and Applied Mathematics
Volume 120 No. 4 2018, 563-572
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v120i4.7

ijpam.eu

BOUNDEDNESS ON UNIFORM SPACES AND IT'S MAPPINGS

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Abstract: In this note we introduce a boundedness on uniform spaces. We study some properties of corresponding bounded sets. So we introduce two types of mappings on uniform spaces from the perspective of the given boundedness. We compare them to each other and with the class of uniformly continuous mappings. Finally we equip these classes of mappings with the topology of uniform convergence induced by the uniformity of uniform convergence. We show some of these classes of mappings form complete uniform spaces provided the range space is complete.

AMS Subject Classification: 54E15, 34C11, 54C35, 42A65

Key Words: uniform space; bounded set; bounded mapping; uniformly continuous mapping; completeness

1. Introduction

Mappings between algebraic structures and mappings between topological structures has always been inspiring new concepts in the areas of mathematics. This factor has led that the mappings study is of particular importance. Much re-

Received: January 26, 2017
Revised: February 15, 2019
Published: February 17, 2018

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url: www.acadpubl.eu

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search has been done on mappings in the topology [1, 9, 12, 17]. Boundedness on topological spaces can not be covered, but this concept about the uniform structures is expressed and investigated. This concept has been studied by many authors [4, 5, 6, 7, 10, 11, 14, 15]. Troitsky [18] and Hejazian et al. [12] have studied some classes of operators on topological vector spaces from the perspective of Von Neumann-boundedness with the help of uniformity caused by the topological vector spaces on the mapping spaces and Kocinac and Zabeti [16] have tried to discuss it on topological groups. Bounded mappings and their uniformity and topological structures are of interest for their own rights and their applications in other areas of mathematics. For examples, bounded operators on a topological vector space with suitable topologies form topological algebras. Also, there is a spectral theory for these classes of bounded operators with some useful applications [13, 18].

In this note we introduce a boundedness on uniform spaces and we study some properties of corresponding bounded sets. Also we study some classes of the mapping spaces between uniform spaces, including *ub*-bounded mappings and *bb*-bounded mappings from the perspective of the given boundedness. We compare them to each other and with the class of uniformly continuous mappings. Finally we equip these classes of mappings with the topology of uniform convergence induced by the uniformity of uniform convergence. We show some of these classes of mappings form complete uniform spaces provided the range space is complete.

Recall that a uniformity on a set X is a non-void family $\mathcal{U}$ of relation on X such that:

(i) Every $U \in \mathcal{U}$ contains the diagonal $\Delta = \{(x, x) : x \in X\}$.

(ii) If $U \in \mathcal{U}$ then $U^{-1} \in \mathcal{U}$.

(iii) If $U \in \mathcal{U}$ then there exists a $V \in \mathcal{U}$ such that $V^2 \subseteq U$.

(iv) If $U \in \mathcal{U}$, $V \in \mathcal{U}$ then $U \cap V \in \mathcal{U}$.

(v) If $U \in \mathcal{U}$, $U \subseteq V \subseteq X \times X$ then $V \in \mathcal{U}$.

Also recall for every uniform space $(X, \mathcal{U})$ there exists a base $\mathcal{B}$ of the uniformity $\mathcal{U}$ with the following properties:

(i) For every $U_1, U_2 \in \mathcal{B}$ there exists a $U \in \mathcal{B}$ such that $U \subseteq U_1 \cap U_2$.

(ii) For every $U \in \mathcal{B}$ there exists a $V \in \mathcal{B}$ such that $V^2 \subseteq U$.

(iii) For every $U \in \mathcal{B}$ and every positive integer n the set U^n is in $\mathcal{B}$.

That can help us in the study of boundedness on uniform spaces and bounded mappings on them.

2. Bounded Sets and Bounded Mappings

In the definition of bounded set on topological vector spaces and on topological groups there are some tools which are absolutely handy: for example, the scaler multiplication, group operation and zero neighborhoods. In general we don't have these tools on an arbitrary uniform space, but instead of scaler multiplication or group operation we can use the concept of combining relation and instead of zero neighborhoods we can use the elements uniformity. In this section based on this description, we introduce a boundedness on uniform spaces. we study some properties of corresponding bounded sets. Also we introduce two types of mappings on uniform spaces from the perspective of the given boundedness. We compare them to each other and with the class of uniformly continuous mappings.

Definition 1. Let $(X,\mathcal{U})$ be a uniform space. We say that a subset B of cartesian product $X \times X$ is *bounded* if for every $U \in \mathcal{U}$ there exists a positive integer n such that $B \subseteq U^n$.

Definition 2. A mapping $f : (X,\mathcal{U}) \to (Y,\mathcal{V})$ is said to be

(i) *ub*-bounded if there exists a $U \in \mathcal{U}$ such that $f \times f(U)$ is bounded in $Y \times Y$.

(ii) *bb*-bounded if for every bounded set B in $X \times X$ the set $f \times f(B)$ is bounded in $Y \times Y$.

(iii) uniformly continuous if for every $V \in \mathcal{V}$ there exists a $U \in \mathcal{U}$ such that $f \times f(U) \subseteq V$.

The class of all *ub*-bounded (*bb*-bounded, uniformly continuous) mappings from a uniform space $(X,\mathcal{U})$ to a uniform space $(Y,\mathcal{V})$ is denoted by $\mathcal{F}_{ub}(X,Y)$ $(\mathcal{F}_{bb}(X,Y), \mathcal{F}_{uc}(X,Y))$. We write $\mathcal{F}(X)$ instead of $\mathcal{F}(X,X)$.

Proposition 3. *For uniform spaces $(X,\mathcal{U})$ and $(Y,\mathcal{V})$ the following holds:*

$$\mathcal{F}_{ub}(X,Y) \subseteq \mathcal{F}_{bb}(X,Y)$$

Proof. Let $f : (X,\mathcal{U}) \to (Y,\mathcal{V})$ be a *ub*-bounded mapping. Then it is *bb*-bounded. Suppose B be a bounded set in $X \times X$. We prove that $f \times f(B)$

is bounded in $Y \times Y$. Since f is ub-bounded, there exists a $U \in \mathcal{U}$ such that $f \times f(U)$ is bounded in $Y \times Y$. Boundedness of B implies that $B \subseteq U^n$ for some natural number n. Now let V be an arbitrary element of $\mathcal{V}$. Boundedness of $f \times f(U)$ implies that $f \times f(U) \subseteq V^n$ for some natural number m. Therefore

$$f \times f(B) \subseteq f \times f(U^n) = ((f \times f(U))^n \subseteq V^{mn}.$$

This implies that $f \times f(B)$ is bounded in $Y \times Y$. □

Proposition 4. *For uniform spaces $(X,\mathcal{U})$ and $(Y,\mathcal{V})$ the following holds:*

$$\mathcal{F}_{uc}(X,Y) \subseteq \mathcal{F}_{bb}(X,Y)$$

Proof. Let $f : (X,\mathcal{U}) \to (Y,\mathcal{V})$ be a uniformly continuous mapping. Then it is bb-bounded. Suppose B be a bounded set in $X \times X$. We prove that $f \times f(B)$ is bounded in $Y \times Y$. Let V be an arbitrary element of $\mathcal{V}$. Since f is uniformly continuous, there exists a $U \in \mathcal{U}$ such that $f \times f(U) \subseteq V$. Boundedness of B implies that $B \subseteq U^n$ for some natural number n. Therefore

$$f \times f(B) \subseteq f \times f(U^n) = ((f \times f(U))^n \subseteq V^n.$$

This implies that $f \times f(B)$ is bounded in $Y \times Y$. □

Definition 5. A uniform space $(X,\mathcal{U})$ is said to be locally bounded if there exists a $U \in \mathcal{U}$ such that U is bounded in $X \times X$.

It's easy to see that for every locally bounded uniform space $(X,\mathcal{U})$, $\mathcal{F}_{ub}(X, Y) = \mathcal{F}_{bb}(X,Y)$. Also it's easy to see that the uniform space $(X,\mathcal{U})$ is locally bounded if and only if the identity mapping $i : (X,\mathcal{U}) \to (X,\mathcal{U})$ is ub-bounded.

Proposition 6. *Let $(X,\mathcal{U})$ be a uniform space and A, B be two bounded sets in $X \times X$. Then the set AB is also bounded.*

Proof. Let U be an arbitrary element of $\mathcal{U}$. Boundedness of A implies That $A \subseteq U^n$ for some natural number n and Boundedness of B implies That $B \subseteq U^m$ for some natural number m. Therefore

$$AB \subseteq U^n U^m = U^{n+m}.$$

This implies that AB is a bounded set in $X \times X$. □

Corollary 7. *Let $(X,\mathcal{U})$ be a locally bounded uniform space and B be a bounded set in $X \times X$. Then there exists a bounded set $U \in \mathcal{U}$ such that $B \subseteq U$.*

The following example shows that the converse of Proposition 2 is not true.

Example 8. Let X be an arbitrary set. For every finite sequence x_1, x_2, ..., x_n of elements of the set X define $V(x_1, x_2, ..., x_n)$ by

$$V(x_1, x_2, ..., x_n) = X \times X - \bigcup_{i=1}^{n} \{(X \times \{x_i\} \cup \{x_i\} \times X)\} - \Delta\}.$$

The sets $V(x_1, x_2, ..., x_n)$ are entourages of the diagonal Δ. Consider $\mathcal{U} \subseteq \mathcal{D}_X$ consisting of all entourages of the diagonal which contain one of the sets $V(x_1, x_2, ..., x_n)$. Clearly $\mathcal{U}$ is a uniformity on the set X. We shall show that the identity mapping $i : (X,\mathcal{U}) \to (X,\mathcal{U})$ is not *ub*-bounded. It's sufficient to show that the uniform space $(X,\mathcal{U})$ is not locally bounded. Let U be an arbitrary element of $\mathcal{U}$. We show U is not bounded. First, consider U contains $V(x_1, x_2, ..., x_n)$ as a proper subset. Since for every positive integer n, $V^n(x_1, x_2, ..., x_n) = V(x_1, x_2, ..., x_n)$, in this case U is not subset of $V^n(x_1, x_2, ..., x_n)$. In the second case consider $U = V(x_1, ..., x_i, ..., x_n) \in \mathcal{U}$. Choose $W(x_1, ..., x_i^{'}, ..., x_n) \in \mathcal{U}$ such that $x_i^{'}$ is different from x_i. Since for every positive integer n, $W^n(x_1, ..., x_i^{'}, ..., x_n) = W(x_1, , ..., x_i^{'}, ..., x_n)$, in this case also U is not subset of $W^n(x_1, ..., x_i^{'}, ..., x_n)$. Therefore the identity mapping $i : (X,\mathcal{U}) \to (X,\mathcal{U})$ is not *ub*-bounded although it's easy to see that i is *bb*-bounded.

Nevertheless, unlike in the case of bounded operators on topological vector space, there is no more relation between uniformly continuous mappings on uniform spaces and bounded ones. The following example shows this fact. Before presenting the example, note that there are many uniform spaces which are bounded. For this purpose, first note to the following lemma:

Lemma 9. [7] *Any totally bounded, connected uniform space is bounded.*

Example 10. Let $\mathcal{U}$ be a uniformity on the set X such that $(X,\mathcal{U})$ be a bounded uniform space. Also consider the set X with the uniformity $\mathcal{U}_c = \{X \times X\}$. Now let $i : (X,\mathcal{U}_c) \to (X,\mathcal{U})$ be the identity mapping. Clearly i is not uniformly continuous. But it is *ub*-bounded as well as *bb*-bounded.

Theorem 11. *Let* $(X,\mathcal{U}) = \Pi_{\alpha \in I}(X_\alpha,\mathcal{U}_\alpha)$ *be the product of the given uniform spaces* $(X_\alpha,\mathcal{U}_\alpha)$. *Then a set* B *in* $X \times X$ *is bounded if and only if for every projection mapping* P_α *the set* $P_\alpha \times P_\alpha(B)$ *is bounded in* $X_\alpha \times X_\alpha$.

Proof. Since the projection mappings of X onto X_α are uniformly continuous, the necessity follows immediately from Proposition 3. We shall show sufficiency. Let B be an arbitrary element of $X \times X$ such that for every $\alpha \in I$, the set $P_\alpha \times P_\alpha(B)$ is bounded in $X_\alpha \times X_\alpha$. Put $B_\alpha := P_\alpha \times P_\alpha(B)$. Since $B \subseteq \Pi_{\alpha\in I} B_\alpha$ it's sufficient to prove the boundedess of $\Pi_{\alpha\in I} B_\alpha$. Let U be an arbitrary element of $\mathcal{U}$. There exist $\alpha_1, \alpha_2, ..., \alpha_k \in I$ and $U_{\alpha_i} \in \mathcal{U}_{\alpha_i}$ such that

$$\{(\{x_{\alpha_i}\}, \{y_{\alpha_i}\}) : (x_{\alpha_i}, y_{\alpha_i}) \in U_{\alpha_i} \ for\ i = 1, 2, ..., k\} \subseteq U.$$

For each $i = 1, 2, ..., k$ there exists a positive integer m_{α_i} such that $B_{\alpha_i} \subseteq U_{\alpha_i}^{m_{\alpha_i}}$. Put $m := \max_{i=1,2,...,k} m_{\alpha_i}$. We prove that $\Pi_{\alpha\in I} B_\alpha \subseteq U^m$. This is clear, if $\alpha \neq \alpha_i$ because for every α different from α_i, $U_\alpha = X_\alpha \times X_\alpha$ and therefore $B_\alpha \subseteq U_\alpha$. Also for each α_i, $B_{\alpha_i} \subseteq U_{\alpha_i}^{m_{\alpha_i}} \subseteq U_\alpha^m$. □

3. Completeness of Mapping Uniform Spases

Let $(X,\mathcal{U})$ and $(Y,\mathcal{V})$ be two uniform spaces. For every $V \in \mathcal{V}$ denote by $\hat{V}$ the entourage of the diagonal $\Delta \subseteq \mathcal{F}(X,Y) \times \mathcal{F}(X,Y)$ defined by the formula

$$\hat{V} = \{(f,g) :\ (f(x), g(x)) \in V \ for\ every\ x \in X\}.$$

Clearly the family $\{\hat{V} : V \in \mathcal{V}\}$ is a base for a uniformity on $\mathcal{F}(X,Y)$. The uniformity on the set $\mathcal{F}(X,Y)$ generated by this family is called the uniformity of uniform convergence induced by $\mathcal{V}$ and denoted by $\hat{\mathcal{V}}$. The topology induced by the uniformity $\hat{\mathcal{V}}$ is called The topology of uniform convergence. We can assume this topology on the family $\mathcal{F}_{ub}(X,Y)$ $(\mathcal{F}_{bb}(X,Y), \mathcal{F}_{uc}(X,Y))$.

In this part, we investigate whether or not each considered classes of mapping in the assumed topology is complete.

Definition 12. A net $\{f_\alpha\}_{\alpha\in I}$ of mappings in the uniform space $(\mathcal{F}(X,Y), \hat{\mathcal{V}})$ is called cauchy net if for every $V \in \mathcal{V}$ there exists $\alpha_0 \in I$ such that for each $\alpha, \beta \geq \alpha_0$. $(f_\alpha(x), f_\beta(x)) \in V$ for every $x \in X$.

Lemma 13. *Let $\{f_\alpha\}_{\alpha\in I}$ be a net of uniformly continuous mappings which converges to the mapping f in the uniform convergence topology. Then f is also a uniformly continuous mapping.*

Proof. Let W be an arbitrary element of $\mathcal{V}$. Choose $V \in \mathcal{V}$ with $V^3 \subseteq W$. Since for each $\alpha \in I$, the mapping f_α is uniformly continuous there exists a $U \in \mathcal{U}$ such that $f \times f(U) \subseteq V$. On the other hand since $f_\alpha \to f$, there exists $\alpha_0 \in I$ such that for each $\alpha \geq \alpha_0$, $(f_\alpha(x), f(x)) \in V$ for every $x \in X$. Fix an $\alpha \geq \alpha_0$. Now for every $(x,y) \in U$,

$$(f(x), f(y)) = (f(x), f_\alpha(x))(f_\alpha(x), f_\alpha(y))(f_\alpha(y), f(y)) \in V^3 \subseteq W.$$

This implies that f is uniformly continuous. □

Lemma 14. *Let $\{f_\alpha\}_{\alpha \in I}$ be a net of bounded mappings which converges to the mappng f in the uniform convergence topology. Then f is also a bb-bounded mapping.*

Proof. Let V be an arbitrary element of $\mathcal{V}$ and B be a bounded set in $X \times X$. Since for each $\alpha \in I$ the mapping f_α is *bb*-bounded, there exists a positive integer n such that $f_\alpha \times f_\alpha(B) \subseteq V^n$. On the other hand since $f_\alpha \to f$, there exists $\alpha_0 \in I$ such that for each $\alpha \geq \alpha_0$, $(f_\alpha(x), f(x)) \in V$ for every $x \in X$. Fix an $\alpha \geq \alpha_0$. Now for every $(x, y) \in B$,

$$(f(x), f(y)) = (f(x), f_\alpha(x))(f_\alpha(x), f_\alpha(y))(f_\alpha(y), f(y)) \in V^{n+2}.$$

This implies that f is *bb*-bounded. □

The class $\mathcal{F}_{ub}(X, Y)$ can contain a cauchy net whose limit is not an *ub*-bounded mapping. In other words, $\mathcal{F}_{ub}(X, Y)$ is not uniformly complete in the assumed topology. The following example shows this fact:

Example 15. Let $X = \{x_1, x_2, ...\}$ be a countable set and $\mathcal{U}$ be the uniformity explained in the Example 8 on X. Consider the sequence of mapping $\{f_n\}_{n \in N}$ as follows:

$$\begin{aligned} f_n(x_1) &= x_1 \\ f_n(x_2) &= x_2 \\ &\vdots \\ f_n(x_n) &= x_n \\ f_n(x_{n+1}) &= x_n \\ f_n(x_{n+2}) &= x_n \\ &\vdots \end{aligned}$$

Each f_n is an *ub*-bounded mapping. For, let $V_n = V(x_1, x_2, ..., x_n) \in \mathcal{U}$ be an entourage of Δ. Then

$$f_n \times f_n(V_n) = \{(x_1, x_1), (x_2, x_2), ..., (x_n, x_n)\} \subseteq \Delta.$$

Since each $V \in \mathcal{V}$ contain Δ, this implies that $f_n \times f_n(V_n)$ is bounded in $X \times X$. On the other hand it's easy to see that the sequence $\{f_n\}_{n \in N}$ is uniformly convergent to the identity mapping i in $\mathcal{F}(X)$. But we have seen in Example 8 that i is not *ub*-bounded.

Theorem 16. *Let* $(X,\mathcal{U})$ *and* $(Y,\mathcal{V})$ *be two uniform spaces and* $(Y,\mathcal{V})$ *be complete. Then* $\mathcal{F}_{uc}(X,Y)$ *is complete with respect to the topology of uniform convergence.*

Proof. Suppose $\{f_\alpha\}_{\alpha\in I}$ be a cauchy net in $\mathcal{F}_{uc}(X,Y)$. Then for every $x\in X$, the net $\{f_\alpha(x)\}_{\alpha\in I}$ is a cauchy net in $(Y,\mathcal{V})$. Since the uniform space $(Y,\mathcal{V})$ is complete, the cauchy net $\{f_\alpha(x)\}_{\alpha\in I}$ is convergent. Put $f(x) := \lim f_\alpha(x)$. by Lemma 13 the mapping f is also uniformly continuous. Let W be an arbitrary element of $\mathcal{V}$. Choose $V\in\mathcal{V}$ with $V^2\subseteq W$. Since $\{f_\alpha(x)\}_{\alpha\in I}$ is a cauchy net in $(Y,\mathcal{V})$, there exists $\alpha_0\in I$ such that for each $\alpha,\beta\geq\alpha_0$, $(f_\alpha(x),f_\beta(x))\in V$ for every $x\in X$. On the other hand for sufficiently large β, $(f_\beta(x),f(x))\in V$. Therefore for each $\alpha\geq\alpha_0$ and $x\in X$,

$$(f_\alpha(x),f(x))=(f_\alpha(x),f_\beta(x))(f_\alpha(x),f(x))\in V^2\subseteq W.$$

This implies that $f_\alpha\to f$. □

Theorem 17. *Let* $(X,\mathcal{U})$ *and* $(Y,\mathcal{V})$ *be two uniform spaces and* $(Y,\mathcal{V})$ *be complete. Then* $\mathcal{F}_{bb}(X,Y)$ *is complete with respect to the topology of uniform convergence.*

Proof. Suppose $\{f_\alpha\}_{\alpha\in I}$ be a cauchy net in $\mathcal{F}_{bb}(X,Y)$. Then for every $x\in X$ the net $\{f_\alpha(x)\}_{\alpha\in I}$ is a cauchy net in $(Y,\mathcal{V})$. Since the uniform space $(Y,\mathcal{V})$ is complete, the cauchy net $\{f_\alpha(x)\}_{\alpha\in I}$ is convergent. Put $f(x) := \lim f_\alpha(x)$. by Lemma 14 the mapping f is also *bb*-bounded. Let W be an arbitrary element of $\mathcal{V}$. Choose $V\in\mathcal{V}$ with $V^2\subseteq W$. Since $\{f_\alpha(x)\}_{\alpha\in I}$ is a cauchy net in $(Y,\mathcal{V})$, there exists $\alpha_0\in I$ such that for each $\alpha,\beta\geq\alpha_0$, $(f_\alpha(x),f_\beta(x))\in V$ for every $x\in X$. On the other hand for sufficiently large β, $(f_\beta(x),f(x))\in V$. Therefore for each $\alpha\geq\alpha_0$ and $x\in X$,

$$(f_\alpha(x),f(x))=(f_\alpha(x),f_\beta(x))(f_\alpha(x),f(x))\in V^2\subseteq W.$$

This implies that $f_\alpha\to f$. □

Note that when the uniform space $(Y,\mathcal{V})$ be complete, then the uniform space $\mathcal{F}_{ub}(X,Y)$ might fail to be complete. For, look Example 8 and Example 15.

Acknowledgment

Authors would like to thank the referee for valuable comments and for careful reading of the manuscript.

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