

RESULTS ON GENERAL ϕ -WEAKLY RANDOM OPERATORS

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Abstract: In this paper, firstly, we prove the existence of random coincidence points for general ϕ -weakly contraction condition under two pairs of random operators, where ϕ is continuous monotone real function. As applications, related common fixed point results are established, the well-posed random fixed point problem is studied and the convergence of random Mann's iteration to a common random fixed point is proved. Our results, essentially, are cover special cases about existence random coincidence points.

AMS Subject Classification: 47B80, 47H40, 60H25

Key Words: p -normed spaces, random coincidence points, common random fixed point, well-posed random fixed point problem

1. Introduction and Preliminaries

The stochastic generalization of coincidence points are random coincidence points. The study of random coincidence point was initiated to Beg and Shahzad [10] who proved important results about common random fixed points and random coincidence points for compatible random operators in Banach

Received: February 24, 2017

Revised: December 6, 2018

Published: February 27, 2018

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url: www.acadpubl.eu

spaces, where compatibility is a generalization of commuting operators. Some random coincidence points for f - non-expansive operators are established using the commutativity condition by Latif and Tweddle [4]. And then, Shahzad and Latif [11] presented other random results of their work. Kumam and Plubtieng [15] proved the existence random coincidence points of compatible single and multivalued random operators and extended the results in [13]. Also, the random coincidence point results are proved in [17] for pair of commuting mapping defined on weakly compact separable subset of complete p -normed space. And then, use them to study the random best approximation in p -normed space with reparability condition. Recently, Gupta and Karapnar [2] introduced the notion of random coupled coincidence points and proved the existence of such points. In [9], Beg and et al. studied random coincidence for weakly compatible random operator which satisfy a weak contraction condition in convex metric spaces . Also, Jhadd and Salua [14] gave other results for multivalued random operator In this field, one can see also [19] and [23]. The aim of this article is to obtain random coincidence point theorem for two pairs of random operators that satisfy contraction condition which is substantially generalization to a condition (1) in [9] (regardless the back ground space X).

Let X be a linear space and $\|\cdot\|_p$ be a real valued function on X with $0 < p \leq 1$. The ordered pair $(X, \|\cdot\|_p)$ is called a p -normed space [15] if for all x, y in X and scalars λ :

- i. $\|x\|_p \geq 0$ and $\|x\|_p = 0$ if and only if $x = 0$.
- ii. $\|\lambda x\|_p = |\lambda|^p \|x\|_p$.
- iii. $\|x + y\|_p \leq \|x\|_p + \|y\|_p$.

for more details about p -normed spaces, see [1] or [3]. Throughout this article X will be separable complete p -normed space whose dual separates the points of it, $\phi \neq A \subseteq X$ be a closed, (Ω, Σ) be the measurable space with Σ a sigma algebra of subsets of Ω , 2^X is the classes of all subsets of X and $CB(X)$ is the classes of all non-empty bounded closed subsets of X .

Definition 1. [16] A mapping $F : \Omega \rightarrow 2^X$ is called measurable (respectively, weakly measurable) if, for any closed (respectively, open) subset B of X ,

$$F^{-1}(B) = \{\omega \in \Omega : F(\omega) \cap B \neq \emptyset\} \in \Sigma$$

Definition 2. [16] A mapping $\delta : \Omega \rightarrow 2^X$ is called a measurable selector of a measurable mapping $F : \Omega \rightarrow 2^X$ if δ measurable and $\delta(\omega) \in F(\omega)$ for each $\omega \in \Omega$.

Definition 3. [14] A mapping $h : \Omega \times X \rightarrow X$ (or $G : \Omega \times X \rightarrow CB(X)$) is called a random operator if for any $x \in X, h(\cdot, x)$ (respectively $G(\cdot, x)$) is

measurable.

Definition 4. [11] A measurable mapping $\delta : \Omega \rightarrow A$ is called random fixed point of a random operator $h : \Omega \times X \rightarrow X$ (or $G : \Omega \times X \rightarrow CB(X)$) if for every $\omega \in \Omega, \delta(\omega) = h(\omega, \delta(\omega))$ (respectively $\delta(\omega) \in G(\omega, \delta(\omega))$).

Definition 5. [9] A measurable mapping $\delta : \Omega \rightarrow A$ is called random coincidence point of a random operator $h : \Omega \times A \rightarrow A$ and $G : \Omega \times A \rightarrow A$ if for every $\omega \in \Omega, h(\omega, \delta(\omega)) = G(\omega, \delta(\omega))$.

Definition 6. [9] A measurable mapping $\delta : \Omega \rightarrow A$ is called common random fixed point of a random operator $h : \Omega \times A \rightarrow X$ and $G : \Omega \times A \rightarrow A$ if for every $\omega \in \Omega$

$$\delta(\omega) = h(\omega, \delta(\omega)) = G(\omega, \delta(\omega))$$

Definition 7. [12] A random operator $h : \Omega \times A \rightarrow X$ is called continuous (weakly continuous) if for each $\omega \in \Omega, h(\omega, \cdot)$ is continuous (weakly continuous).

Now, we define a new type of random operators.

Definition 8. Let $h, G, S, T : \Omega \times X \rightarrow X$ be four random operators. (h, G, ϕ) is called generalized weakly contractive with respect to the pair (S, T) if for all $x, y \in X$,

$$\|S(\omega, x) - T(\omega, y)\|_p \leq M(x, y) - \phi(M(x, y)) \tag{1}$$

where:

$$M(x, y) = \max \left\{ \|h(\omega, x) - G(\omega, y)\|_p, \|h(\omega, x) - S(\omega, x)\|_p, \|G(\omega, y) - T(\omega, y)\|_p, \frac{1}{2} \left[\|h(\omega, x) - T(\omega, y)\|_p + \|G(\omega, y) - h(\omega, x)\|_p \right] \right\}$$

for each $x, y \in \Omega$, where $\phi : [0, \infty) \rightarrow [0, \infty)$ is continuous and nondecreasing map such that, $\phi(0) = 0$ and $\lim_{t \rightarrow \infty} \phi(t) = \infty$.

As special case is:

Definition 9. Let $h, S : \Omega \times X \rightarrow X$ be two random operators. Then (h, ϕ) is called weakly contractive with respect to S if for all $x, y \in X$,

$$\|S(\omega, x) - S(\omega, y)\|_p \leq \|h(\omega, x) - h(\omega, y)\|_p - \phi \left(\|h(\omega, x) - h(\omega, y)\|_p \right) \tag{2}$$

for each $x, y \in \Omega$, where $\phi : [0, \infty) \rightarrow [0, \infty)$ is continuous and nondecreasing map such that, $\phi(0) = 0$ and $\lim_{t \rightarrow \infty} \phi(t) = \infty$.

Definition 10. [7] Let $h, G : X \rightarrow X$ be two mappings, then h and G are called R -weakly commuting if for all $x \in X$ there exists $R > 0$, such that:

$$\|Ghx - hGx\|_p \leq R \|Gx - hx\|_p$$

Definition 11. [8] A pair (h, G) of self mappings of X is said to be weakly compatible, if they commute at their coincidence points, i.e., $hGx = Ghx$ for all x satisfying $h(x) = G(x)$.

The following definition appeared in [7] and [8] respectively:

Definition 12. A random operators $h, G : \Omega \times X \rightarrow X$ are said to be R -weakly commute (or Weakly Compatible) if $h(\omega, \cdot)$ and $G(\omega, \cdot)$ are R -weakly commute (respectively weakly compatible) for each $\omega \in \Omega$.

2. Random Coincidence Theorems

We prove that:

Theorem 13. Let $h, G, S, T : \Omega \times A \rightarrow A$ be random operators and the pairs (S, T) and (h, G) satisfy condition (1). If for each $\omega \in \Omega$, $S(\omega, A) \subseteq G(\omega, A)$, $T(\omega, A) \subseteq h(\omega, A)$ and one of the subset $S(\omega, A)$, $h(\omega, A)$, $T(\omega, A)$ or $G(\omega, A)$ is a separable complete subspace of A . Then: i. The pair S, h has random coincidence point; ii. The pair T, G has random coincidence point.

Proof. Let $\delta_0 : \Omega \rightarrow A$ be arbitrary measurable mapping. Set $y_0 = S(\omega, \delta_0(\omega))$. We construct a sequence of measurable mappings $\delta_n : \Omega \rightarrow A$ as the following: Since $S(\omega, A) \subseteq G(\omega, A)$, then we can find $\delta_1 : \Omega \rightarrow A$, such that $y_0 = S(\omega, \delta_0(\omega)) = G(\omega, \delta_1(\omega))$. Set $y_1 = T(\omega, \delta_1(\omega))$. Since $T(\omega, A) \subseteq h(\omega, A)$, then there exists $\delta_2 : \Omega \rightarrow A$, such that $h(\omega, \delta_2(\omega)) = T(\omega, \delta_1(\omega)) = y_1$. By induction, we have two sequences $\{y_n\}$ and $\{\delta_n\}$ in A , such that for all nonnegative integer

$$y_{2n} = S(\omega, \delta_{2n}(\omega)) = G(\omega, \delta_{2n+1}(\omega)) \quad (3)$$

and

$$y_{2n+1} = h(\omega, \delta_{2n+2}(\omega)) = T(\omega, \delta_{2n+1}(\omega)) \quad (4)$$

From (3), (4) and (1), we have:

$$\begin{aligned}
 \|y_{2n+2} - y_{2n+1}\|_p &= \|S(\omega, \delta_{2n+2}(\omega)) - T(\omega, \delta_{2n+1}(\omega))\|_p \\
 &\leq M(\delta_{2n+2}(\omega), \delta_{2n+1}(\omega)) - \phi(M(\delta_{2n+2}(\omega), \delta_{2n+1}(\omega))) \\
 &= \max \left\{ \|h(\omega, \delta_{2n+2}(\omega)) - G(\omega, \delta_{2n+1}(\omega))\|_p, \|h(\omega, \delta_{2n+2}(\omega)) - \right. \\
 &\quad S(\omega, \delta_{2n+2}(\omega))\|_p, \|G(\omega, \delta_{2n+1}(\omega)) - T(\omega, \delta_{2n+1}(\omega))\|_p, \\
 &\quad \left. \frac{1}{2} \left[\|h(\omega, \delta_{2n+2}(\omega)) - T(\omega, \delta_{2n+1}(\omega))\|_p + \|G(\omega, \delta_{2n+1}(\omega)) - \right. \right. \\
 &\quad \left. \left. S(\omega, \delta_{2n+2}(\omega))\|_p \right] \right\} - \phi \left(\left\{ \|h(\omega, \delta_{2n+2}(\omega)) - G(\omega, \delta_{2n+1}(\omega))\|_p, \right. \right. \\
 &\quad \left. \left. \|h(\omega, \delta_{2n+2}(\omega)) - S(\omega, \delta_{2n+2}(\omega))\|_p, \|G(\omega, \delta_{2n+1}(\omega)) - \right. \right. \\
 &\quad \left. \left. T(\omega, \delta_{2n+1}(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, \delta_{2n+2}(\omega)) - T(\omega, \delta_{2n+1}(\omega))\|_p + \right. \right. \right. \\
 &\quad \left. \left. \left. \|G(\omega, \delta_{2n+1}(\omega)) - S(\omega, \delta_{2n+2}(\omega))\|_p \right] \right\} \right) \\
 &= \max \left\{ \|y_{2n+1} - y_{2n}\|_p, \|y_{2n+1} - y_{2n+2}\|_p, \|y_{2n} - y_{2n+1}\|_p, \right. \\
 &\quad \left. \frac{1}{2} \left[\|y_{2n+1} - y_{2n+1}\|_p + \|y_{2n} - y_{2n+2}\|_p \right] \right\} - \\
 &\quad \phi \left(\max \left\{ \|y_{2n+1} - y_{2n}\|_p, \|y_{2n+1} - y_{2n+2}\|_p, \|y_{2n} - y_{2n+1}\|_p, \right. \right. \\
 &\quad \left. \left. \frac{1}{2} \left[\|y_{2n+1} - y_{2n+1}\|_p + \|y_{2n} - y_{2n+2}\|_p \right] \right\} \right) \\
 &= \max \left\{ \|y_{2n+1} - y_{2n}\|_p, \|y_{2n+1} - y_{2n+2}\|_p, \frac{1}{2} \|y_{2n} - y_{2n+2}\|_p \right\} - \\
 &\quad \phi \left(\max \left\{ \|y_{2n+1} - y_{2n}\|_p, \|y_{2n+1} - y_{2n+2}\|_p, \frac{1}{2} \|y_{2n} - y_{2n+2}\|_p \right\} \right)
 \end{aligned} \tag{5}$$

Using triangle inequality, property $\frac{1}{2} [a + b] \leq \max\{a, b\}$ and property of ϕ , we get:

$$\begin{aligned}
 \|y_{2n+2} - y_{2n+1}\|_p &\leq \max \left\{ \|y_{2n+1} - y_{2n}\|_p, \|y_{2n+1} - y_{2n+2}\|_p, \frac{1}{2} \left[\|y_{2n} - \right. \right. \\
 &\quad \left. \left. y_{2n+1}\|_p + \|y_{2n+1} - y_{2n+2}\|_p \right] \right\} - \phi \left(\max \left\{ \|y_{2n+1} - y_{2n}\|_p, \right. \right. \\
 &\quad \left. \left. \|y_{2n+1} - y_{2n+2}\|_p, \frac{1}{2} \left[\|y_{2n} - y_{2n+1}\|_p + \|y_{2n+1} - y_{2n+2}\|_p \right] \right\} \right) \\
 &\leq \max \left\{ \|y_{2n+1} - y_{2n}\|_p, \|y_{2n+1} - y_{2n+2}\|_p \right\} - \phi \left(\max \left\{ \|y_{2n+1} - \right. \right. \\
 &\quad \left. \left. y_{2n}\|_p, \|y_{2n+1} - y_{2n+2}\|_p \right\} \right)
 \end{aligned} \tag{6}$$

If $\max \left\{ \|y_{2n+1} - y_{2n}\|_p, \|y_{2n+1} - y_{2n+2}\|_p \right\} = \|y_{2n+1} - y_{2n+2}\|_p$, then by (6), we have

$$\begin{aligned}
 0 &< \|y_{2n+1} - y_{2n+2}\|_p \leq \|y_{2n+1} - y_{2n+2}\|_p - \phi \left(\|y_{2n+1} - y_{2n+2}\|_p \right) \\
 &< \|y_{2n+1} - y_{2n}\|_p,
 \end{aligned}$$

which is a contradiction. Thus,

$\max \{ \|y_{2n+1} - y_{2n}\|_p, \|y_{2n+1} - y_{2n+2}\|_p \} = \|y_{2n+1} - y_{2n}\|_p$. This implies,

$$\begin{aligned} \|y_{2n+1} - y_{2n+2}\|_p &\leq \|y_{2n+1} - y_{2n}\|_p - \phi(\|y_{2n+1} - y_{2n}\|_p) \\ &\leq \|y_{2n+1} - y_{2n}\|_p. \end{aligned}$$

Hence for all nonnegative integer n , we have:

$$\|y_{2n+1} - y_{2n+2}\|_p \leq \|y_{2n+1} - y_{2n}\|_p$$

thus sequence $\{ \|y_{2n+1} - y_{2n+2}\|_p \}$ is non-increasing of positive real numbers. Hence, it converges to $r \geq 0$. If $r > 0$, then:

$$\|y_n - y_{n+1}\|_p \leq \|y_{n-1} - y_n\|_p - \phi(\|y_{n-1} - y_n\|_p)$$

$r \leq r - \theta(r) < r$, which is contradiction. Hence $r = 0$ and

$$\lim_{n \rightarrow \infty} \|y_{n+1} - y_n\|_p = 0 \quad (7)$$

Now, we show $\{y_n\}$ is a Cauchy sequence. Since $\lim_{n \rightarrow \infty} \|y_{n+1} - y_n\|_p = 0$, then we need only to show that $\{y_{2n}\}$ is a Cauchy sequence. Suppose that this is not the case, then there exists $\varepsilon > 0$ and there exists even integers $2n(k)$ and $2m(k)$ with $2k \leq 2m(k) < 2n(k)$, such that:

$$\|y_{2m(k)} - y_{2n(k)}\|_p > \varepsilon \text{ and } \|y_{2n(k)-2} - y_{2m(k)}\|_p \leq \varepsilon$$

Using (7) following inequality, we have:

$$\varepsilon < \|y_{2m(k)} - y_{2n(k)}\|_p \leq \|y_{2m(k)} - y_{2n(k)-2}\|_p + \|y_{2n(k)-2} - y_{2n(k)-1}\|_p + \|y_{2n(k)-1} - y_{2n(k)}\|_p$$

hence $\varepsilon < \lim_{k \rightarrow \infty} \|y_{2m(k)} - y_{2n(k)}\|_p \leq \varepsilon + 0 + 0$. Therefore:

$$\lim_{k \rightarrow \infty} \|y_{2m(k)} - y_{2n(k)}\|_p = \varepsilon \quad (8)$$

Also (7), (8) following inequality, we have:

$$\|y_{2m(k)} - y_{2n(k)}\|_p \leq \|y_{2m(k)} - y_{2m(k)+1}\|_p + \|y_{2m(k)+1} - y_{2n(k)}\|_p$$

Then:

$$\lim_{k \rightarrow \infty} \|y_{2m(k)} - y_{2n(k)}\|_p \leq \lim_{k \rightarrow \infty} \|y_{2m(k)} - y_{2m(k)+1}\|_p + \lim_{k \rightarrow \infty} \|y_{2m(k)+1} - y_{2n(k)}\|_p$$

$$\leq 0 + \lim_{k \rightarrow \infty} \|y_{2m(k)+1} - y_{2n(k)}\|_p$$

while (5), (7) and inequality:

$$\begin{aligned} \|y_{2m(k)+1} - y_{2n(k)}\|_p &\leq \|y_{2m(k)+1} - y_{2n(k)}\|_p + \|y_{2m(k)} - y_{2n(k)}\|_p \\ \lim_{k \rightarrow \infty} \|y_{2m(k)+1} - y_{2n(k)}\|_p &\leq 0 + \varepsilon \end{aligned}$$

Hence, $\lim_{k \rightarrow \infty} \|y_{2m(k)+1} - y_{2n(k)}\|_p \leq 0 + \varepsilon$, and this implies:

$$\lim_{k \rightarrow \infty} \|y_{2m(k)+1} - y_{2n(k)}\|_p = \varepsilon \tag{9}$$

By the similar way, we have:

$$\lim_{k \rightarrow \infty} \|y_{2m(k)} - y_{2n(k)-1}\|_p = \lim_{k \rightarrow \infty} \|y_{2n(k)-1} - y_{2m(k)+1}\|_p = \varepsilon \tag{10}$$

for all nonnegative integer k , (1) implies that:

$$\begin{aligned} \|y_{2n(k)} - y_{2m(k)+1}\|_p &= \|S(\omega, y_{2n(k)}) - T(\omega, y_{2m(k)+1})\|_p \\ &\leq M(y_{2n(k)}, y_{2m(k)+1}) - \phi(My_{2n(k)}, y_{2m(k)+1}) \\ &\leq \|y_{2n(k)-1} - y_{2m(k)}\|_p - \phi\left(\|y_{2n(k)-1} - y_{2m(k)}\|_p\right) \\ \lim_{k \rightarrow \infty} \|y_{2n(k)} - y_{2m(k)+1}\|_p &\leq \lim_{k \rightarrow \infty} \|y_{2n(k)-1} - y_{2m(k)}\|_p - \\ &\quad \lim_{k \rightarrow \infty} \phi\left(\|y_{2n(k)-1} - y_{2m(k)}\|_p\right) \end{aligned}$$

Also, (9) and (10), implies that $\varepsilon \leq \varepsilon - \phi(\varepsilon)$, which is contradiction with $\varepsilon > 0$, hence $\{y_{2n}\}$ is a Cauchy sequence. Suppose that $G(\omega, A)$ is a complete subspace of , this implies the sequence $\{y_n\}$ has a limit $t : \Omega \rightarrow G(A)$. We obtained a mapping $u : \Omega \rightarrow A$, such that $(\omega, u(\omega)) = t(\omega)$. So:

$$t(\omega) = \lim_{n \rightarrow \infty} y_{2n} = \lim_{n \rightarrow \infty} S(\omega, \delta_{2n}(\omega)) = \lim_{n \rightarrow \infty} G(\omega, \delta_{2n+1}(\omega))$$

and

$$\begin{aligned} t(\omega) &= \lim_{n \rightarrow \infty} y_{2n+1} = \lim_{n \rightarrow \infty} h(\omega, \delta_{2n+2}(\omega)) = \lim_{n \rightarrow \infty} T(\omega, \delta_{2n+1}(\omega)) \\ &= \lim_{n \rightarrow \infty} G(\omega, \delta_{2n+1}(\omega)) \end{aligned}$$

Using (3) and (1), we have:

$$\begin{aligned} \|y_{2n} - T(\omega, u(\omega))\|_p &= \|S(\omega, \delta_{2n}(\omega)) - T(\omega, u(\omega))\|_p \\ &\leq \max \left\{ \|h(\omega, \delta_{2n}(\omega)) - G(\omega, u(\omega))\|_p, \|h(\omega, \delta_{2n}(\omega)) - S(\omega, \delta_{2n}(\omega))\|_p, \right. \\ &\quad \|G(\omega, u(\omega)) - T(\omega, u(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, \delta_{2n}(\omega)) - T(\omega, u(\omega))\|_p + \right. \\ &\quad \left. \|G(\omega, u(\omega)) - S(\omega, \delta_{2n}(\omega))\|_p \right] \left. \right\} - \phi \left(\max \left\{ \|h(\omega, \delta_{2n}(\omega)) - \right. \right. \\ &\quad \left. G(\omega, u(\omega))\|_p, \|h(\omega, \delta_{2n}(\omega)) - \delta_{2n}(\omega, u(\omega))\|_p, \|G(\omega, u(\omega)) - \right. \\ &\quad \left. T(\omega, u(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, \delta_{2n}(\omega)) - T(\omega, u(\omega))\|_p + \|G(\omega, u(\omega)) - \right. \right. \\ &\quad \left. \left. S(\omega, \delta_{2n}(\omega))\|_p \right] \right\} \right) \end{aligned}$$

taking limit as $n \rightarrow \infty$, we get:

$$\begin{aligned} \|t(\omega) - T(\omega, u(\omega))\|_p &\leq \max \left\{ \|t(\omega) - T(\omega, u(\omega))\|_p, \frac{1}{2} \|t(\omega) - \right. \\ &\quad \left. T(\omega, u(\omega))\|_p \right\} - \phi \left(\max \left\{ \|G(\omega, u(\omega)) - T(\omega, u(\omega))\|_p, \frac{1}{2} \|t(\omega) - \right. \right. \\ &\quad \left. \left. T(\omega, u(\omega))\|_p \right\} \right) \\ &= \|t(\omega) - T(\omega, u(\omega))\|_p - \phi \left(\|t(\omega) - T(\omega, u(\omega))\|_p \right) \end{aligned}$$

by properties of ϕ , obtaining $\phi \left(\|t(\omega) - T(\omega, u(\omega))\|_p \right) = 0$, this implies

$$\|t(\omega) - T(\omega, u(\omega))\|_p = 0,$$

and hence:

$$t(\omega) = T(\omega, u(\omega)) = G(\omega, u(\omega)) \tag{11}$$

Since $(\omega, A) \subseteq h(\omega, A)$, then $t(\omega) \in h(\omega, A)$, a mapping $v : \Omega \rightarrow A$, exists, such that:

$$h(\omega, v(\omega)) = t(\omega) \tag{12}$$

By (11), (1) and (12), we have:

$$\|S(\omega, v(\omega)) - t(\omega)\|_p = \|S(\omega, v(\omega)) - T(\omega, u(\omega))\|_p$$

$$\begin{aligned}
 &\leq \max \left\{ \|h(\omega, v(\omega)) - G(\omega, u(\omega))\|_p, \|h(\omega, v(\omega)) - S(\omega, v(\omega))\|_p, \right. \\
 &\quad \|G(\omega, u(\omega)) - T(\omega, u(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, v(\omega)) - T(\omega, u(\omega))\|_p + \right. \\
 &\quad \left. \left. \|G(\omega, u(\omega)) - S(\omega, v(\omega))\|_p \right] \right\} - \phi \left(\max \{ \|h(\omega, v(\omega)) - \right. \\
 &\quad G(\omega, u(\omega))\|_p, \|h(\omega, v(\omega)) - S(\omega, v(\omega))\|_p, \|G(\omega, u(\omega)) - \\
 &\quad T(\omega, u(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, v(\omega)) - T(\omega, u(\omega))\|_p + \right. \\
 &\quad \left. \left. \|G(\omega, u(\omega)) - S(\omega, v(\omega))\|_p \right] \right\} \right) \\
 &= \max \left\{ \|t(\omega) - S(\omega, v(\omega))\|_p, \frac{1}{2} \|t(\omega) - S(\omega, v(\omega))\|_p \right\} - \\
 &\quad \phi \left(\max \left\{ \|t(\omega) - S(\omega, v(\omega))\|_p, \frac{1}{2} \|t(\omega) - S(\omega, v(\omega))\|_p \right\} \right) \\
 &= \|t(\omega) - S(\omega, v(\omega))\|_p - \phi \left(\|t(\omega) - S(\omega, v(\omega))\|_p \right)
 \end{aligned}$$

so we get $\phi \left(\|t(\omega) - S(\omega, v(\omega))\|_p \right) = 0$. This implies $\|t(\omega) - S(\omega, v(\omega))\|_p = 0$. Therefore, $h(\omega, v(\omega)) = h(\omega, v(\omega)) = t(\omega)$. Hence $v : \Omega \rightarrow A$ is a random coincidence point of hand S . Finally, we have:

$$h(\omega, v(\omega)) = h(\omega, v(\omega)) = t(\omega) = T(\omega, u(\omega)) = G(\omega, u(\omega)) \tag{13}$$

Theorem 14. *Let $S, T, h, G : \Omega \times A \rightarrow A$ satisfying inequality (1). If the pairs $\{S, h\}$ and $\{T, G\}$ are weakly compatible (R -weakly commuting), $S(\omega, A) \subseteq G(\omega, A)$, $T(\omega, A) \subseteq h(\omega, A)$ and one of the subsets $S(\omega, A)$, $h(\omega, A)$, $T(\omega, A)$ or $G(\omega, A)$ is a separable complete subspace of A , then S, h, T and G have a unique common random fixed point.*

Proof. By theorem (13), there exists random coincidence point $u : \Omega \rightarrow A$ of G and T , such that $T(\omega, u(\omega)) = G(\omega, u(\omega))$ and random coincidence point of G and S , such that $h(\omega, v(\omega)) = S(\omega, v(\omega))$. If the pairs $\{S, h\}$ and $\{T, G\}$ are weakly compatible, then:

$$S(\omega, h(\omega, v(\omega))) = h(\omega, S(\omega, v(\omega)))$$

and

$$T(\omega, G(\omega, u(\omega))) = G(\omega, T(\omega, u(\omega)))$$

from (13), we have:

$$S(\omega, t(\omega)) = h(\omega, t(\omega)) \text{ and } T(\omega, t(\omega)) = G(\omega, t(\omega)) \tag{14}$$

from (13), (1) and (14), we have:

$$\begin{aligned}
 \|t(\omega) - T(\omega, t(\omega))\|_p &= \|S(\omega, v(\omega)) - T(\omega, t(\omega))\|_p \\
 &\leq \max \left\{ \|h(\omega, v(\omega)) - G(\omega, t(\omega))\|_p, \|h(\omega, v(\omega)) - S(\omega, t(\omega))\|_p, \right. \\
 &\quad \|G(\omega, t(\omega)) - T(\omega, t(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, v(\omega)) - T(\omega, t(\omega))\|_p + \right. \\
 &\quad \left. \left. \|G(\omega, t(\omega)) - S(\omega, v(\omega))\|_p \right] \right\} - \phi \left(\max \{ \|h(\omega, v(\omega)) - \right. \\
 &\quad G(\omega, t(\omega))\|_p, \|h(\omega, v(\omega)) - S(\omega, v(\omega))\|_p, \|G(\omega, t(\omega)) - \\
 &\quad T(\omega, t(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, v(\omega)) - T(\omega, t(\omega))\|_p + \|G(\omega, t(\omega)) - \right. \\
 &\quad \left. \left. S(\omega, v(\omega))\|_p \right] \right\} \right) \\
 &= \|t(\omega) - T(\omega, t(\omega))\|_p - \phi \left(\|t(\omega) - T(\omega, t(\omega))\|_p \right)
 \end{aligned}$$

This implies $\phi \left(\|t(\omega) - T(\omega, t(\omega))\|_p \right) = 0$. Thus by properties of ϕ , we have $\|t(\omega) - T(\omega, t(\omega))\|_p = 0$. Therefore $t(\omega) = T(\omega, t(\omega))$. From (14), we have:

$$t(\omega) = T(\omega, t(\omega)) = G(\omega, t(\omega)) \quad (15)$$

Again, from (15), (1) and (14), we get:

$$\begin{aligned}
 \|S(\omega, t(\omega)) - t(\omega)\|_p &= \|S(\omega, t(\omega)) - T(\omega, t(\omega))\|_p \\
 &\leq \max \left\{ \|h(\omega, t(\omega)) - G(\omega, t(\omega))\|_p, \|h(\omega, t(\omega)) - S(\omega, t(\omega))\|_p, \right. \\
 &\quad \|G(\omega, t(\omega)) - T(\omega, t(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, t(\omega)) - T(\omega, t(\omega))\|_p + \right. \\
 &\quad \left. \left. \|G(\omega, t(\omega)) - S(\omega, t(\omega))\|_p \right] \right\} - \phi \left(\max \{ \|h(\omega, t(\omega)) - \right. \\
 &\quad G(\omega, t(\omega))\|_p, \|h(\omega, t(\omega)) - S(\omega, t(\omega))\|_p, \|G(\omega, t(\omega)) - \\
 &\quad T(\omega, t(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, t(\omega)) - T(\omega, t(\omega))\|_p + \right. \\
 &\quad \left. \left. \|G(\omega, t(\omega)) - S(\omega, t(\omega))\|_p \right] \right\} \right)
 \end{aligned}$$

$$= \|S(\omega, t(\omega)) - t(\omega)\|_p - \phi \left(\|S(\omega, t(\omega)) - t(\omega)\|_p \right)$$

Hence $\phi \left(\|S(\omega, t(\omega)) - t(\omega)\|_p \right) = 0$, then by property of ϕ and (14), we have:

$$S(\omega, t(\omega)) = t(\omega) = h(\omega, t(\omega)) \tag{16}$$

Since $T(\omega, t(\omega)) = G(\omega, t(\omega)) = t(\omega)$, then:

$$S(\omega, t(\omega)) = h(\omega, t(\omega)) = T(\omega, t(\omega)) = G(\omega, t(\omega)) = t(\omega) \tag{17}$$

Thus $t : \Omega \rightarrow G(A)$ is a common random fixed point of S, T, h and G .

Uniqueness: Let $\alpha(\omega)$ be another common random fixed point of S, T, h and G , then by using (1), we have:

$$\begin{aligned} \|t(\omega) - \alpha(\omega)\|_p &= \|S(\omega, t(\omega)) - T(\omega, \alpha(\omega))\|_p \\ &\leq \max \left\{ \|h(\omega, t(\omega)) - G(\omega, \alpha(\omega))\|_p, \|h(\omega, t(\omega)) - S(\omega, t(\omega))\|_p, \right. \\ &\quad \|G(\omega, \alpha(\omega)) - T(\omega, \alpha(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, t(\omega)) - T(\omega, \alpha(\omega))\|_p + \right. \\ &\quad \left. \left. \|G(\omega, \alpha(\omega)) - S(\omega, t(\omega))\|_p \right] \right\} - \phi \left(\max \left\{ \|h(\omega, t(\omega)) - \right. \right. \\ &\quad \left. \left. G(\omega, \alpha(\omega))\|_p, \|h(\omega, t(\omega)) - S(\omega, t(\omega))\|_p, \|G(\omega, \alpha(\omega)) - \right. \right. \\ &\quad \left. \left. T(\omega, \alpha(\omega))\|_p, \frac{1}{2} \left[\|h(\omega, t(\omega)) - T(\omega, \alpha(\omega))\|_p + \right. \right. \right. \\ &\quad \left. \left. \left. \|G(\omega, \alpha(\omega)) - S(\omega, t(\omega))\|_p \right] \right\} \right) \\ &= \|t(\omega) - \alpha(\omega)\|_p - \phi \left(\|t(\omega) - \alpha(\omega)\|_p \right) \end{aligned}$$

Which gives $\phi \left(\|t(\omega) - \alpha(\omega)\|_p \right) = 0$, hence by properties of ϕ , we have

$$\|t(\omega) - \alpha(\omega)\|_p = 0,$$

which implies to $t(\omega) = \alpha(\omega)$ Assume that $\{S, h\}$ is R -weakly commuting and $v : \Omega \rightarrow A$ is a random coincidence point of S and h , it follows that:

$$\|S(\omega, h(\omega, v(\omega))) - h(\omega, S(\omega, v(\omega)))\|_p \leq R \|G(\omega, v(\omega)) - h(\omega, v(\omega))\|_p = 0$$

Then $S(\omega, h(\omega, v(\omega))) - h(\omega, S(\omega, v(\omega))) = 0$, and thus $S(\omega, h(\omega, v(\omega))) = h(\omega, S(\omega, v(\omega)))$ Hence the pair $\{S, G\}$ is weakly compatible.

Similarly, we have $\{T, h\}$ is weakly compatible, then the same steps above we can show that $t : \Omega \rightarrow h(A)$ is a unique common random fixed point of S , T , h and G .

As a consequence, we get the following:

Corollary 15. *Let $S, h : \Omega \times A \rightarrow A$ and for all $x, y \in A$*

$$\begin{aligned} \|S(\omega, x) - S(\omega, y)\|_p \leq & \max \left\{ \|h(\omega, x) - h(\omega, y)\|_p, \|h(\omega, x) - S(\omega, x)\|_p, \right. \\ & \|h(\omega, y) - S(\omega, y)\|_p, \frac{1}{2} \left[\|h(\omega, x) - S(\omega, y)\|_p + \right. \\ & \left. \|h(\omega, y) - S(\omega, x)\|_p \right] \left. \right\} - \phi \left(\max \left(\|h(\omega, x) - h(\omega, y)\|_p, \right. \right. \\ & \left. \|h(\omega, x) - S(\omega, x)\|_p, \|h(\omega, y) - S(\omega, y)\|_p \right. \\ & \left. \left. \frac{1}{2} \left[\|h(\omega, x) - S(\omega, y)\|_p + \|h(\omega, y) - S(\omega, x)\|_p \right] \right) \right) \end{aligned} \quad (18)$$

If $S(\omega, A) \in h(\omega, A)$ and one of the subset $S(\omega, A)$ or $h(\omega, A)$ is a separable complete p -normed subspace of A . Then the pair $\{S, h\}$ has random coincidence point.

Corollary 16. *Let $S, h : \Omega \times A \rightarrow A$ satisfying inequality (19). If the pair $\{S, h\}$ is weakly compatible (R -weakly commuting), $S(\omega, A) \in h(\omega, A)$ and one of the subset $S(\omega, A)$ or $h(\omega, A)$ is a separable complete p -normed subspace of A , then the pair $\{S, h\}$ has unique common random fixed point.*

Corollary 17. *Let $S, h : \Omega \times A \rightarrow A$, such that h is ϕ -weakly contractive with respect to S . If $S(\omega, A) \in h(\omega, A)$ and one of the subset $S(\omega, A)$ or $h(\omega, A)$ is a separable complete p -normed subspace of . Then the pair $\{S, h\}$ has random coincidence point.*

Corollary 18. *Let $S, h : \Omega \times A \rightarrow A$, such that h is ϕ -weakly contractive with respect to S . If the pair $\{S, h\}$ is weakly compatible (R -weakly commuting), $S(\omega, A) \in h(\omega, A)$ and one of the subset $S(\omega, A)$ or $h(\omega, A)$ is a separable complete p -normed subspace of , then the pair $\{S, h\}$ has unique common random fixed point.*

Here, we must refer to analogous result appeared recently in [20] and [21].

3. Random Iteration Theorem

We define the following:

Definition 19. Let $h, S : \Omega \times X \rightarrow X$ be two random operators, such that h is ϕ -weakly contractive with respect to S on a separable complete p -normed space X , and $S(\omega, X) \in h(\omega, X)$ with $h(\omega, X)$ convex subset of X , a sequence:

$$y_n = h(\omega, x_{n+1}) = (1 - \alpha_n(\omega))h(\omega, x_n) + \alpha_n S(\omega, x_n), \quad x \in X, \quad n \geq 0$$

where $\alpha_n : \Omega \rightarrow [0, 1]$ for each $n \in N$ is called random Mann iterative scheme.

Theorem 20. Let $S, h : \Omega \times A \rightarrow A$, such that h is ϕ -weakly contractive with respect to S . If the pair $\{S, h\}$ is weakly compatible or R -weakly commuting and $(\omega, A) \in h(\omega, A)$, with $h(\omega, A)$ as a convex separable complete subspace of A , then random Mann iterative scheme with $\sum_{n=1}^{\infty} \alpha_n(\omega) = \infty$, converges to common random fixed point of S and h .

Proof. By Corollary 18, S and h have a unique common random fixed point $: \Omega \rightarrow h(\omega, A)$, such that:

$$h(\omega, t(\omega)) = S(\omega, t(\omega)) = t(\omega) \text{ and } t(\omega) = h(\omega, u(\omega)) = S(\omega, u(\omega))$$

Now, let $\{\delta_n(\omega)\}$ be the random Mann iterative scheme defined in Definition 19, then:

$$\begin{aligned} \|\delta_n(\omega) - t(\omega)\|_p &= \|h(\omega, \delta_{n+1}(\omega)) - h(\omega, u(\omega))\|_p \\ &= \|(1 - \alpha_n(\omega))h(\omega, \delta_n(\omega)) + \alpha_n(\omega)S(\omega, \delta_n(\omega)) - h(\omega, u(\omega))\|_p \\ &= \|(1 - \alpha_n(\omega))h(\omega, \delta_n(\omega)) + \alpha_n(\omega)S(\omega, \delta_n(\omega)) - h(\omega, u(\omega)) + \\ &\quad \alpha_n(\omega)h(\omega, u(\omega)) - \alpha_n(\omega)h(\omega, u(\omega))\|_p \\ &= \|(1 - \alpha_n(\omega))(h(\omega, \delta_n(\omega)) - h(\omega, u(\omega))) + \alpha_n(\omega)S(\omega, \delta_n(\omega)) - \\ &\quad S(\omega, u(\omega))\|_p \\ &\leq |1 - \alpha_n(\omega)|^p \|h(\omega, \delta_n(\omega)) - h(\omega, u(\omega))\|_p + \\ &\quad |\alpha_n(\omega)|^p \|S(\omega, \delta_n(\omega)) - S(\omega, u(\omega))\|_p \\ &\leq (1 - \alpha_n(\omega)) \|h(\omega, \delta_n(\omega)) - h(\omega, u(\omega))\|_p + \end{aligned}$$

$$\alpha_n(\omega) \|S(\omega, \delta_n(\omega)) - S(\omega, u(\omega))\|_p$$

Since h is ϕ -weakly contractive with respect to S , then:

$$\begin{aligned} \|\delta_n(\omega) - t(\omega)\|_p &\leq (1 - \alpha_n(\omega)) \|h(\omega, \delta_n(\omega)) - h(\omega, u(\omega))\|_p + \alpha_n(\omega) \|h(\omega, \delta_n(\omega)) - \\ &\quad h(\omega, u(\omega))\|_p - \alpha_n(\omega) \phi \left(\|h(\omega, \delta_n(\omega)) - h(\omega, u(\omega))\|_p \right) \\ &\leq (1 - \alpha_n(\omega)) \|h(\omega, \delta_n(\omega)) - h(\omega, u(\omega))\|_p + \\ \alpha_n(\omega) \|h(\omega, \delta_n(\omega)) - h(\omega, u(\omega))\|_p \end{aligned}$$

Thus:

$$\|\delta_n(\omega) - t(\omega)\|_p \leq \|h(\omega, \delta_n(\omega)) - h(\omega, u(\omega))\|_p = \|\delta_{n-1}(\omega) - t(\omega)\|_p$$

thus the sequence $\{\|\delta_n(\omega) - t(\omega)\|_p\}$ is non-increasing of positive real numbers. Hence, it converges to $r \geq 0$. If $r > 0$, then for any positive integer N , we have:

$$\begin{aligned} \sum_{n=N}^{\infty} \alpha_n(\omega) \phi(\omega, r) &\leq \sum_{n=N}^{\infty} \alpha_n(\omega) \phi \left(\|\delta_n(\omega) - t(\omega)\|_p \right) \\ &\leq \sum_{n=N}^{\infty} \left[\|\delta_n(\omega) - t(\omega)\|_p - \|\delta_{n+1}(\omega) - t(\omega)\|_p \right] \\ &\leq \sum_{n=N}^{\infty} \|\delta_N(\omega) - t(\omega)\|_p \end{aligned}$$

which contradicts $\sum_{n=1}^{\infty} \alpha_n(\omega) = \infty$, hence random Mann iterative scheme converges to common random fixed point of S and h .

4. Random Well-posed Problem

Several researchers have been study the well-posedness of a fixed point problem for single/multivalued mappings in the usual metric spaces, for examples [5], [6], [18], [23]. Hence we extend the notion of well-posedness to random fixed point problem.

Definition 21. Let $(X, \|\cdot\|_p)$ be a p -normed space and $T : \Omega \times X \rightarrow X$ a random operator. The random fixed point problem of T is said to be well-posed if:

i. T has a unique random fixed point $\delta : \Omega \rightarrow X$.

ii. For any sequence $\{\delta_n(\omega)\}$ of measurable mappings in X , such that:

$$\lim_{n \rightarrow \infty} \|T(\omega, \delta_n(\omega)) - \delta_n(\omega)\|_p = 0$$

we have:

$$\lim_{n \rightarrow \infty} \|\delta_n(\omega) - \delta(\omega)\|_p = 0$$

Definition 22. Let $(X, \|\cdot\|_p)$ be a p -normed space and let \mathcal{T} be a set of a random operators in X . The random fixed point of \mathcal{T} is said to be well-posed if:

- i. T has a unique random fixed point $\delta : \Omega \rightarrow X$.
- ii. For any sequence $\{\delta_n(\omega)\}$ of measurable mappings in X , such that:

$$\lim_{n \rightarrow \infty} \|T(\omega, \delta_n(\omega)) - \delta_n(\omega)\|_p = 0, \forall T \in \mathcal{T}$$

we have:

$$\lim_{n \rightarrow \infty} \|\delta_n(\omega) - \delta(\omega)\|_p = 0$$

Theorem 23. Let X be a p -normed space and $\phi \neq A \subseteq X$ and S, T, h, G be a self random operators on A satisfying inequality (1). If the pairs $\{S, h\}$ and $\{G, T\}$ are weakly compatible or R -weakly commuting, $S(\omega, A) \subseteq G(\omega, A)$, $T(\omega, A) \subseteq h(\omega, A)$, for all $\omega \in \Omega$ and one of the subsets $S(\omega, A), T(\omega, A), G(\omega, A)$ or $h(\omega, A)$ is a separable complete subspace of A , then the common random fixed point for the set of random mappings $\{S, T, h, G\}$ is well-posed.

Proof. By Theorem 20, the random mappings S, T, h and G have a unique common random fixed point $t : \Omega \rightarrow A$. Let $\{\delta_n(\omega)\}$ be a sequence of measurable mappings in A , such that:

$$\begin{aligned} \lim_{n \rightarrow \infty} \|S(\omega, \delta_n(\omega)) - \delta_n(\omega)\|_p &= \lim_{n \rightarrow \infty} \|T(\omega, \delta_n(\omega)) - \delta_n(\omega)\|_p \\ &= \lim_{n \rightarrow \infty} \|h(\omega, \delta_n(\omega)) - \delta_n(\omega)\|_p \\ &= \lim_{n \rightarrow \infty} \|G(\omega, \delta_n(\omega)) - \delta_n(\omega)\|_p = 0 \end{aligned}$$

For all nonnegative integer n , we have:

$$M(t(\omega), \delta_n(\omega)) = \max \left\{ \|h(\omega, t(\omega)) - G(\omega, \delta_n(\omega))\|_p, \|h(\omega, t(\omega)) - \right.$$

$$\begin{aligned} & S(\omega, t(\omega))\|_p, \|G(\omega, \delta_n(\omega)) - T(\omega, \delta_n(\omega))\|_p, \\ & \|h(\omega, t(\omega)) - T(\omega, \delta_n(\omega))\|_p + \|G(\omega, \delta_n(\omega)) - \\ & h(\omega, t(\omega))\|_p \Big\} \end{aligned}$$

from (17), property $\frac{1}{2}(a + b) \leq \max\{a, b\}$ and using the triangle inequality, we get:

$$\begin{aligned} M(t(\omega), \delta_n(\omega)) &= \max \left\{ \|t(\omega) - G(\omega, \delta_n(\omega))\|_p, \|G(\omega, \delta_n(\omega)) - \right. \\ & \left. T(\omega, \delta_n(\omega))\|_p, \|t(\omega) - T(\omega, \delta_n(\omega))\|_p \right\} \\ &\leq \max \left\{ \|t(\omega) - \delta_n(\omega)\|_p + \|\delta_n(\omega) - G(\omega, \delta_n(\omega))\|_p, \right. \\ & \|G(\omega, t(\omega)) - t(\omega)\|_p + \|\delta_n(\omega) - T(\omega, \delta_n(\omega))\|_p, \\ & \left. \|h(\omega, t(\omega)) - \delta_n(\omega)\|_p + \|\delta_n(\omega) - T(\omega, \delta_n(\omega))\|_p \right\} \\ &\leq \|t(\omega) - \delta_n(\omega)\|_p + \|\delta_n(\omega) - G(\omega, \delta_n(\omega))\|_p + \\ & \quad \|\delta_n(\omega) - T(\omega, \delta_n(\omega))\|_p \end{aligned} \tag{19}$$

By the triangle inequality, (17), (1) and inequality (19), we have:

$$\begin{aligned} \|t(\omega) - \delta_n(\omega)\|_p &\leq \|t(\omega) - T(\omega, \delta_n(\omega))\|_p + \|T(\omega, \delta_n(\omega)) - \delta_n(\omega)\|_p \\ &= \|S(\omega, t(\omega)) - T(\omega, \delta_n(\omega))\|_p + \|T(\omega, \delta_n(\omega)) - \delta_n(\omega)\|_p \\ \phi(M(t(\omega), \delta_n(\omega))) &\leq \|\delta_n(\omega) - G(\omega, \delta_n(\omega))\|_p + 2\|\delta_n(\omega) - T(\omega, \delta_n(\omega))\|_p \end{aligned} \tag{20}$$

Thus, we have:

$$\lim_{n \rightarrow \infty} \phi(M(t(\omega), \delta_n(\omega))) = 0 \tag{21}$$

To get contradiction, let $\{\delta_n(\omega)\}$ does not converge to $t(\omega)$. Then there exists a positive number $\varepsilon > 0$ and subsequence $\{\delta_m(\omega)\}$, such that:

$$\|t(\omega) - G(\omega, \delta_m(\omega))\|_p \geq \varepsilon, \text{ for all integer } m$$

Since ϕ is nondecreasing, from (20) and (21), we have:

$$\begin{aligned}\phi(\varepsilon) &\leq \phi\left(\|t(\omega) - G(\omega, \delta_m(\omega))\|_p\right) \\ &\leq \phi(M(t(\omega), G(\omega, \delta_m(\omega)))) \\ &\leq \phi\left(\|\delta_m(\omega) - G(\omega, (\omega))\|_p + 2\|\delta_m(\omega) - T(\omega, (\omega))\|_p\right)\end{aligned}$$

By letting $m \rightarrow \infty$, we get $\phi(\varepsilon) = 0$, a contradiction to the property ϕ . Thus $\lim_{n \rightarrow \infty} \|\delta_n(\omega) - t(\omega)\|_p = 0$

Corollary 24. *Let $\emptyset \neq A$ separable complete subspace of X , and S and T be a self random mappings on A satisfying the following condition:*

$$\begin{aligned}\|S(\omega, x) - T(\omega, y)\|_p &\leq \max\left\{\|x - y\|_p, \|x - S(\omega, x)\|_p, \|y - T(\omega, y)\|_p, \right. \\ &\quad \left. \frac{1}{2}\left[\|x - T(\omega, y)\|_p + \|y - S(\omega, x)\|_p\right]\right\} - \phi\left(\max\left\{\|x - y\|_p, \right. \right. \\ &\quad \left. \|x - S(\omega, x)\|_p, \|y - T(\omega, y)\|_p, \frac{1}{2}\left[\|x - T(\omega, y)\|_p + \right. \right. \\ &\quad \left. \left. \|y - S(\omega, x)\|_p\right]\right\}\end{aligned}$$

where $\phi : [0, \infty) \rightarrow [0, \infty)$ is lower semi-continuous function with $\phi(t) > 0$, for all $t \in [0, \infty)$ and $\phi(0) = 0$. Then, there exists a unique common random fixed point $\delta : \Omega \rightarrow A$ of S and T .

Moreover, if ϕ is nondecreasing function then the common random fixed point for the pair $\{S, T\}$ is well-posed.

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