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WHITE-NOISE FUNCTIONAL SOLUTIONS FOR WICK-TYPE STOCHASTIC TIME-FRACTIONAL BENJAMIN-BONA-MAHONY EQUATION

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Abstract: A multiple families of White-noise functional solutions for Wick-type stochastic time-fractional Benjamin-Bona-Mahony (BBM) equations modified with variable coefficients are proposed. The suggested technique is based on employing white noise analysis, Hermite and inverse Hermite transforms, improved homogeneous balance method and a modified tanh-coth method; to explore some exact solutions for time-fractional BBM equation with variable coefficients in random processes. Rigorous computations with implemented formal examples are explicitly introduced.

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1. Introduction

The goal of this paper is to explore exact solutions for the following time-fractional Benjamin-Bona-Mahony equation with variable coefficients:

$$u_{t^\alpha} + u_x + p(t)uu_x - q(t)u_{xxt^\alpha} = 0, \quad u_{t^\alpha} := D_t^\alpha u = \frac{\partial^\alpha u}{\partial t^\alpha}, \tag{1.1}$$

where $p(t)$, $q(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$ and D_t^α is the fractional derivative in the Caputo's sense [1-3]. The standard Benjamin-Bona-Mahony equation with $\alpha = 1, \quad p(t) = 1$ and $q(t) = 1$; plays a significant role in many scientific applications such as fluid mechanics, plasma physics, solid state physics, nonlinear optics and chemical kinetics, etc. Benjamin, Bona and Mahoney [4] advocated that the PDE modeled the same physical phenomena equally well as the KdV equation, given the same assumptions and approximations that were originally used by Korteweg and de Vries [5]. This PDE of Benjamin et al. [4] is now often called the BBM equation, although it is also known as the regularized long wave (RLW) equation. Morrison et al. [6] proposed the one-dimensional PDE, as an equally valid and accurate model for the same wave phenomena simulated by the KdV and RLW equations -Random waves is an important subject of random PDEs. In essence, to investigate the exact solutions of random Benjamin-Bona-Mahony equation, we restrict our attention to consider this problem in white noise environment [7-11]. In [12-14], Wadati studies the stochastic KdV equation and gives the diffusion of solition of the KdV equation under Gaussian noise. Xie firstly researched Wick-type stochastic KdV equation on white noise space; he introduced the auto-Bächlund transformation and the exact white noise functional solutions in [15], recently developed by Wei et al [16] and Gurefe et al [17]. Furthermore, Chen [18-20] and Xie [21-23] researched some Wick-type stochastic wave equations using white noise analysis method. The innovator work of Malfiet in [24-26] presented the powerful tanh method for a dependable treatment of the nonlinear wave equation. Recently, the extended tanh method, developed by Wazwaz [27-28], is a valuable technique for handling nonlinear equations. Later, among Uğurlu and Kaya [29], Wazzan [30], Jabbari and Kheiri [31] proposed various extensions of the tanh-coth method. These valuable methods have been extensively applied

to derive nonlinear transformations and exact solutions of nonlinear PDEs in mathematical physics, e.g. see [14, 18, 27-29].

Definition. The Riemann-Liouville fractional integral operator (J^α) with order $\alpha \geq 0$, of a function $h \in C_\mu$, $\mu \geq -1$, is defined as

$$\begin{aligned} J^\alpha h(t) &= \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} h(\tau) d\tau \quad (\alpha > 0), \\ J^0 h(t) &= h(t), \end{aligned} \tag{1.2}$$

$\Gamma(\alpha)$ is the well- known Gamma function.

Some basic properties of the operator J^α, which we will need here, are as follows:

(1) $J^\alpha J^\beta h(t) = J^{\alpha+\beta} h(t)$,

(2) $J^\alpha J^\beta h(t) = J^\beta J^\alpha h(t)$,

(3) $J^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} t^{\alpha+\gamma}$, where $\beta \geqslant 0$, and $\gamma \geqslant -1$.

Definition. The fractional derivative (D^α) of $h(t)$ in the Caputo's sense is defined as

$$\begin{aligned} D^\alpha h(t) &= \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} h^{(n)}(\tau) d\tau, \\ \text{for } n-1 &< \alpha \leq n, \quad n \in \mathbb{N}\ , t > 0, \quad h \in C_{-1}^n. \end{aligned} \tag{1.3}$$

The Caputo's fractional derivative has the following two essential properties:

(1) Let $h \in C_{-1}^n$, $n \in \mathbb{N}$. Then $D^\alpha h$, $0 \leq \alpha \leq n$ is well defined and $D^\alpha h \in C_{-1}$.

(2) Let $n-1 < \alpha \leq n$, $n \in \mathbb{N}$ and $h \in C_\mu^n$, $\mu \geq -1$. Then

$$(J^\alpha D^\alpha) h(t) = h(t) - \sum_{k=0}^{n-1} h^{(k)}(0^+) \frac{t^k}{k!}. \tag{1.4}$$

The core idea of this work is to consider Eq. (1.1) in random versions; this in turn results in a random fractional Benjamin-Bona-Mahony equation. And in order to deduce the exact solutions of such random fractional BBM equations, we shall investigate it in white noise environment sense only. If all products and all functions of Eq. (1.1) are interpreted as their Wick-type, then Eq. (1.1) can be transformed into the following generalized Wick-type stochastic fractional BBM equations in white noise environment:

$$U_{t^\alpha} + U_x + P(t) \diamond U \diamond U_x - Q(t) \diamond U_{xxt^\alpha} = 0, \tag{1.5}$$

where " $\diamond$"is the Wick product on the Kondratiev distribution space $\mathcal{S}_{-1}$ as defined in [7], P(t) and Q(t) are $\mathcal{S}_{-1}$valued functions. To handle this problem, it is convenient to use Hermite transform which converts Wick product into ordinary (complex) products on space $\mathcal{S}_{-1}$, see Holden et al [28]. Under certain transformation for the independent variable , and the use of the homogeneous balance principle together with the aid of Maple, the exact white noise functional solutions for the Wick-type stochastic form of the variable coefficient time-fractional BBM equations are obtained.

2. Initial Structure and Main Results

This section is dedicated to establish the main structure for deducing the white-noise functional solution of Eq. (1.5) which is the perturbation of the time-fractional BBM equation with variable coefficients (1.1). Hermite Transform of Eq. (1.5) turns the Wick products of (1.5) into the following regular products:

$$\widetilde{U}_{t^\alpha}(x,t,z) + \widetilde{U}_x(x,t,z) + \widetilde{P}(t,z)\widetilde{U}\widetilde{U}_x(x,t,z) - \widetilde{Q}(t,z)\widetilde{U}_{xxt^\alpha}(x,t,z) = 0 \tag{2.1}$$

where $\widetilde{U}, \widetilde{P}, \widetilde{Q}$ are the Hermite transforms of U, P, Q respectively and $z = (z_1, z_2, ...) \in \mathbb{C}^{\mathbb{N}}$ is a vector complex parameter i.e, we are seeking for a solution $u(x,t,z)$ of Eq.(2.1) for each z belongs to the space

$$\mathbb{K}_m(n) = \{z \in \mathbb{C}^n; \qquad \sum_{k\neq 0} |z^k|^2 (2\mathbb{N})^{mk} < n^2, \quad n, m > 0\}$$

Using the transformation :

$$\tau := \frac{\mu t^\alpha}{\Gamma(1+\alpha)}$$

that will carry out eqn. (2.1) into

$$\mu\widetilde{U}_\tau(x,\tau,z) + \widetilde{U}_x(x,\tau,z) + \widetilde{P}(\tau,z)\widetilde{U}\widetilde{U}_x(x,\tau,z) - \mu\widetilde{Q}(\tau,z)\widetilde{U}_{xx\tau}(x,\tau,z) = 0 \tag{2.2}$$

Setting $u(\tau,x,z) = \widetilde{U}(\tau,x,z), p(\tau,z) = \widetilde{P}(\tau,z), q(\tau,z) = \widetilde{Q}(\tau,z)$ and $r(\tau,z) = \widetilde{R}(\tau,z)$. This equation can be written as

$$\mu u_\tau(\tau,x,z)u_x(\tau,x,z)+p(\tau,z)u(\tau,x,z)u_x(\tau,x,z) - \mu q(\tau,z)u_{xx\tau}(\tau,x,z)=0 \quad (2.3)$$

Under the assumption that the solution of Eq.(2.3) can be rewritten as

$$u(\tau,x,z)=(\phi\circ\zeta)(\tau,x,z) \quad (2.4)$$

with

$$\zeta(\tau,x,z)=kx+s\int_0^\tau l(\nu,z)d\nu+c$$

where k, s, c are arbitrary constants which satisfy $ks\neq 0$ and $l(\nu,z)$ nonzero functions of indicated variables to be determined. Threat, $u(\tau,x,z)$ is replaced by $\phi(\zeta)$ which defines the solution of Eq.(2.3). So, a direct substitution of Eq.(2.4) into Eq.(2.3) reduces the PDE in Eq.(2.3) which can be integrated once, providing that all its terms contain derivatives, where the integration constants are considered zero. This yields the following second order differential equation

$$[k+\mu sl(\tau,z)]\phi(\zeta)+\frac{1}{2}kp(\tau,z)\phi^2(\zeta)-k^2q(\tau,z)\mu sl(\tau,z)\frac{d^2\phi}{d\zeta^2}=0, \quad (2.5)$$

Considering the homogeneous balance between $\frac{d^2\phi}{d\zeta^2}$ and ϕ^2 in (2.5) [30], gives $M=2$, hence $\phi(x,\tau,z)$ can be expressed by the modified tanh-coth method as:

$$\phi(x,\tau,z)=S(Y)=a_0(\tau,z)+a_1(\tau,z)Y(\zeta)+a_2(\tau,z)Y^2(\zeta)+b_1(\tau,z)Y^{-1}(\zeta) + b_2(\tau,z)Y^{-2}(\zeta) \quad (2.6)$$

where $Y(\zeta)$ satisfies the Riccati equation

$$Y'=c_0+c_1Y+c_2Y^2, \quad (2.7)$$

and c_0,c_1,c_2 are constant to be prescribed later. Substituting (2.6) with (2.7) into Eq. (2.5), taking into account the linear independence of Y^n ($n=-4,-3,$

..., 4), resulting the following system of nine equations

$$\begin{cases} (k+\mu sl)a_0 + \frac{1}{2}kp(a_0^2 + 2a_1b_1 + 2a_2b_2) - \mu k^2 qsl\{c_0(a_1c_1 + 2a_2c_0) \\ \qquad + c_2(b_1c_1 + 2b_2c_2)\} = 0, \\ (k+\mu sl)a_1 + \frac{1}{2}kp(2a_0a_1 + 2a_2b_1) \\ \qquad - \mu k^2 qsl\{c_1(a_1c_1 + 6a_2c_0) + 2c_0a_1c_2\} = 0, \\ (k+\mu sl)a_2 + \frac{1}{2}kp(a_1^2 + 2a_0a_2) \\ \qquad - \mu k^2 qsl\{c_1(3a_1c_2 + 4a_2c_1) + 8c_0a_2c_2\} = 0, \\ (k+\mu sl)b_1 + \frac{1}{2}kp(2a_0b_1 + 2a_1b_2) \\ \qquad - \mu k^2 qsl\{c_1(b_1c_1 + 6b_2c_2) + 2c_0b_1c_2\} = 0, \\ (k+\mu sl)b_2 + \frac{1}{2}kp(b_1^2 + 2a_0b_2) \\ \qquad - \mu k^2 qsl\{c_1(3b_1c_0 + 4b_2c_1) + 8c_0b_2c_2\} = 0, \\ kpa_1a_2 - 2\mu k^2 qslc_2(a_1c_2 + 5a_2c_1) = 0, \\ kpb_1b_2 - 2\mu k^2 qslc_0(b_1c_0 + 5b_2c_1) = 0, \\ \frac{1}{2}kpa_2^2 - 6\mu k^2 qsla_2c_2^2 = 0, \\ \frac{1}{2}kpb_2^2 - 6\mu k^2 qslb_2c_0^2 = 0. \end{cases} \tag{2.8}$$

In what follows, we explore and solve Eq. (2.3) for some particular wide cases of the Riccati Eq. (2.7). In the remaining part of this section we investigate and solve our problem for some particular cases for the Riccati equation (2.7) as follows:

Case 1: For c_0 and $c_1 = c_2 = \lambda$ the Riccati equation has specific solutions for any nonzero value of λ given by

$$Y_1(\zeta) = \pm\{exp(-\lambda\zeta) - 1\}^{-1} \tag{2.9}$$

Using Maple to solve the resulting system (2.8) for nonzero terms p and q, we obtain $b_1 = b_2 = 0, \pm a_1 = a_2 = \frac{12}{p}\lambda^2 kq\mu sl, p \neq 0, q \neq 0, l = \frac{-k}{\mu s}(1 + \lambda^2 k^2 q)^{-1}$ and $a_0 = \frac{-2}{p}\{1 + \frac{\mu sl}{k}\}$

Substituting these values with Eq.(2.9) in Eq.(2.6) we get the following particular solution of Eq.(2.3):

$$u_1(\tau, x, z) = \frac{-\lambda^2 k^2 q(\tau, z)}{[1 + \lambda^2 k^2 q(\tau, z)]p(\tau, z)}[2 + 3csch^2(\frac{\lambda}{2}\zeta(\tau, x, z))] \tag{2.10}$$

with

$$\zeta(\tau, x, z) = \frac{k}{\mu}[x\mu - \int_0^\tau (1 + \lambda^2 k^2 q(\nu, z))^{-1} d\nu] + c.$$

Case 2: For $c_2 = 0$ and $c_1 = \pm c_0 = \lambda \neq 0$ the Riccati equation has specific solutions for any nonzero value of λ given by

$$Y_2(\zeta) = \frac{1}{\lambda}\{exp(\lambda\zeta) \mp \lambda\} \tag{2.11}$$

Solve the resulting system (2.8) for nonzero terms p and q, we get the following coefficients $a_1 = a_2 = 0, b_1 = \pm b_2 = \pm\frac{12}{p}\lambda^2 kq\mu sl, p \neq 0, q \neq 0, l = \frac{-k}{\mu s}(1 + \lambda^2 k^2 q)^{-1}$ and $a_0 = \frac{-2}{p}\{1 + \frac{\mu sl}{k}\}$.

Substituting these values with $Y_1(\zeta) = \frac{1}{\lambda}\{exp(\lambda\zeta) \mp \lambda\}$ in Eq.(2.6) we get the following particular solution of Eq.(2.3):

$$u_{2,3}(\tau, x, z) = \frac{-\lambda^2 k^2 q(\tau, z)}{[1 + \lambda^2 k^2 q(\tau, z)]p(\tau, z)}\{\frac{1 + 6\lambda\zeta(\tau, x, z)}{exp[\lambda\zeta(\tau, x, z)] \pm \lambda}\} \tag{2.12}$$

with

$$\zeta(\tau, x, z) = \frac{k}{\mu}[x\mu - \int_0^\tau (1 + \lambda^2 k^2 q(\nu, z))^{-1} d\nu] + c.$$

Case 3-A: Setting $c_1 = 0$ and $c_0 = c_2 = \lambda \neq 0$ the Riccati equation has the following solutions

$$Y_3(\zeta) = tan(\lambda\zeta) \quad Y_4(\zeta) = -cot(\lambda\zeta) \tag{2.13}$$

Solve the resulting system (2.8) for nonzero terms p and q, gives the following coefficients $a_1 = b_1 = 0, a_2 = b_2 = \frac{12}{p}\lambda^2 kq\mu sl, p \neq 0, q \neq 0, l = \frac{-k}{\mu s}(1 + 16\lambda^2 k^2 q)^{-1}$ and $a_0 = \frac{-1}{p}\{1 + \frac{\mu sl}{k} - 8\lambda^2 kq\mu sl\}$. Recalling these values with (2.12) in Eq.(2.6) we get the following particular solution of Eq.(2.3):

$$u_4(\tau, x, z) = \frac{-48\lambda^2 k^2 q(\tau, z)}{[1 + 16\lambda^2 k^2 q(\tau, z)]p(\tau, z)}\{csc^2(2\lambda\zeta(\tau, x, z))\} \tag{2.14}$$

with

$$\zeta(\tau, x, z) = \frac{k}{\mu}[x\mu - \int_0^\tau (1 + 16\lambda^2 k^2 q(\nu, z))^{-1} d\nu] + c.$$

And

$$u_5(\tau, x, z) = \frac{-16\lambda^2 k^2 q(\tau, z)}{[1 - 16\lambda^2 k^2 q(\tau, z)]p(\tau, z)}\{-2 + 3csc^2(2\lambda\zeta(\tau, x, z))\} \tag{2.15}$$

with

$$\zeta(\tau, x, z) = \frac{k}{\mu}[x\mu - \int_0^\tau (1 - 16\lambda^2 k^2 q(\nu, z))^{-1} d\nu] + c.$$

Case 3-B: Setting $c_1 = 0$ and $c_0 = -c_2 = \lambda \neq 0$ the Riccati equation has the following solutions

$$Y_5(\zeta) = tanh(\lambda\zeta) \quad Y_6(\zeta) = -coth(\lambda\zeta) \tag{2.16}$$

Solve the resulting system (2.8) for nonzero terms p and q, gives the following coefficients $a_1 = b_1 = 0, a_2 = b_2 = \frac{12}{p}\lambda^2 kq\mu sl, p \neq 0, q \neq 0, l = \frac{-k}{\mu s}(1 + 16\lambda^2 k^2 q)^{-1}$ and $a_0 = \frac{-1}{p}\{1 + \frac{\mu sl}{k} + 8\lambda^2 kq\mu sl\}$. Recalling these values with (2.12) in Eq.(2.6) we get the following particular solution of Eq.(2.3):

$$u_6(\tau, x, z) = \frac{-48\lambda^2 k^2 q(\tau, z)}{[1 - 16\lambda^2 k^2 q(\tau, z)]p(\tau, z)}\{csch^2(2\lambda\zeta(\tau, x, z))] \tag{2.17}$$

with

$$\zeta(\tau, x, z) = \frac{k}{\mu}[x\mu - \int_0^\tau (1 - 16\lambda^2 k^2 q(\nu, z))^{-1} d\nu] + c.$$

And

$$u_7(\tau, x, z) = \frac{-16\lambda^2 k^2 q(\tau, z)}{[1 + 16\lambda^2 k^2 q(\tau, z)]p(\tau, z)}\{2 + 3csch^2(2\lambda\zeta(\tau, x, z))] \tag{2.18}$$

with

$$\zeta(\tau, x, z) = \frac{k}{\mu}[x\mu - \int_0^\tau (1 + 16\lambda^2 k^2 q(\nu, z))^{-1} d\nu] + c.$$

Evidently, there are infinitely number of particular solutions for system (2.8) together with Riccati equation (2.7) resulting in many different cases. By virtue of the introduced nonzero parameter λ , the above mentioned cases covering a huge number of specific solutions for (2.8) showing how far our suggested technique is applicable.

3. White Noise Functional Solutions of (2.1)

In order to establish white noise functional solutions of Eqs.(2.1), we make use Theorem 2.1 of Xie [7] for $d = 1$. The properties of hyperbolic functions yield that there exists a bounded open set $\mathbf{S} \subset \mathbb{R}_+ \times \mathbb{R}, m > 0$ and $n > 0$ such that $u(x, t, z), u_{xt}(x, t, z)$ are uniformly bounded for all $(t, x, z) \in \mathbf{S} \times \mathbb{K}_m(n)$, continuous with respect to $(t, x) \in \mathbf{S}$ for all $z \in \mathbb{K}_m(n)$ and analytic with respect to $z \in \mathbb{K}_m(n)$ for all $(t, x) \in \mathbf{S}$. Threat, theorem 2.1 of Xie [7], assures the

existence of a stochastic process $U(t,x)$ such that the Hermite transformation of $U(t,x)$ is $u(t,x,z)$ for all $\mathbf{S}\times\mathbb{K}_m(n)$, and thereby, $U(t,x)$ is the solution of (2.1). This implies that $U(t,x)$ is the inverse Hermite transformation of $u(t,x,z)$. Consequently, for $P(t)Q(t)\neq 0$ the white noise functional solutions of Eqs.(2.1) as follows:

$$U_1(\tau,x,z)=\frac{-\lambda^2k^2Q(\tau,z)}{[1+\lambda^2k^2Q(\tau,z)]P(\tau,z)}[2+3csch^{\diamond 2}(\frac{\lambda}{2}\Xi_1(\tau,x,z))] \tag{3.1}$$

and

$$U_{2,3}(\tau,x,z)=\frac{-\lambda^2k^2Q(\tau,z)}{[1+\lambda^2k^2Q(\tau,z)]P(\tau,z)}\{\frac{1+6\lambda\Xi_1(\tau,x,z)}{exp^{\diamond}[\lambda\Xi_1(\tau,x,z)]\pm\lambda}\} \tag{3.2}$$

with

$$\Xi_1(\tau,x,z)=\frac{k}{\mu}[x\mu-\int_0^{\tau}(1+\lambda^2k^2Q(\nu,z))^{-1}d\nu]+c.$$

$$U_4(\tau,x,z)=\frac{-48\lambda^2k^2Q(\tau,z)}{[1+16\lambda^2k^2Q(\tau,z)]P(\tau,z)}\{csc^{\diamond 2}(2\lambda\Xi_2(\tau,x,z))] \tag{3.3}$$

and

$$U_5(\tau,x,z)=\frac{-16\lambda^2k^2Q(\tau,z)}{[1-16\lambda^2k^2Q(\tau,z)]P(\tau,z)}\{-2+3csc^{\diamond 2}(2\lambda\Xi_2(\tau,x,z))] \tag{3.4}$$

with

$$\Xi_2(\tau,x,z)=\frac{k}{\mu}[x\mu-\int_0^{\tau}(1-16\lambda^2k^2Q(\nu,z))^{-1}d\nu]+c.$$

$$U_6(\tau,x,z)=\frac{-48\lambda^2k^2Q(\tau,z)}{[1-16\lambda^2k^2Q(\tau,z)]P(\tau,z)}\{csch^{\diamond 2}(2\lambda\Xi_3(\tau,x,z))] \tag{3.5}$$

with

$$\Xi_3(\tau,x,z)=\frac{k}{\mu}[x\mu-\int_0^{\tau}(1-16\lambda^2k^2Q(\nu,z))^{-1}d\nu]+c.$$

and

$$U_7(\tau,x,z)=\frac{-16\lambda^2k^2Q(\tau,z)}{[1+16\lambda^2k^2Q(\tau,z)]P(\tau,z)}\{2+3csch^{\diamond 2}(2\lambda\Xi_4(\tau,x,z))] \tag{3.6}$$

with

$$\Xi_4(\tau,x,z)=\frac{k}{\mu}[x\mu-\int_0^{\tau}(1+16\lambda^2k^2Q(\nu,z))^{-1}d\nu]+c.$$

4. Concluding Remarks

The above results leads us to conclude that many kinds of exact solutions to the considered BBM equation with variable coefficients (1.1) can be obtained by following the suggested technique which is mainly based on employing white noise analysis, Hermite transform, homogeneous balance principle and the modified tanh-coth method. Obviously, this technique can be easily extended to cover many other similar nonlinear wave equations. The obtained solutions include solitary wave solutions and periodic wave solutions. Significantly, the proposed technique changes the partial differential equation with variable coefficients to solving ordinary differential equation - Riccati equation. Clearly, the used Riccati equation posses many other particular solutions [31], depending on the chosen values of c_0, c_1 and c_2 , this in turn results many other exact solutions for the considered wick-type stochastic BBM equations. . Moreover, Since there is a unitary map between the Wiener and the Poisson white noise spaces, we can obtain the solution of the Poissonian SPDE simply applying this map to the solution of the corresponding Gaussian SPDE. Hence, we can get solutions as we did in Section 2 if the coefficients $p(t)$ and $q(t)$ are Poissonian white noise functionals in (1.1). Finally, we did not include some other formal solutions on more general nonlinear partial differential equations although we have partial results along these lines. We defer the study of such equation to future work, for more details see [32].

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