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WATER TABLE FLUCTUATIONS DUE TO TIME VARYING RECHARGE IN A 1-D FLOW SYSTEM FROM RECHARGE BASIN

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Abstract: The numerical solution of a 1-D linearized Boussinesq's equation is presented to predict the water table fluctuations in an unconfined aquifer due to time varying recharge from recharge (rectangular) basin for an one canal. The solution is obtained by using finite difference method. Characteristic behaviours of the solution are illustrated with the help of a numerical example to predict the water table fluctuations.

Key Words: linearized Boussinesq's equation, time varying recharge, unconfined aquifer, recharge (rectangular) basin, canal, finite difference method

1. Introduction

The numerical models are widely used for the analysis of basin scale groundwater

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hydrology and to quantify the water exchange between different aquifer levels [6]. Numerical methods yield approximate solutions to the governing equation through the discretisation of space and time.

In the finite difference method the partial differential equation representing groundwater flow is replaced by a set of difference equations which are solved numerically to yield values of the unknown dependent variable at the nodes within the region of investigation.

Maasland (1959)has investigated the fluctuation of water table in a finite aquifer due to a transient recharge. He has approximated the transient recharge pattern in terms of piecewise constant rate of recharge and used the results of constant rate of recharge to get the results for the transient recharge. Marino (1974) has developed analytical solutions for the rise, decline and maximum height of the water table in a finite aquifer receiving vertical recharge.

Rai and Singh (1985) have studied a general analytical solution in dimensionless form is developed to describe the fluctuation of water table in a finite aquifer in response to a transient recharge. The rate of recharge is considered as decreasing linearly with time to a lower value and thereafter remains constant. Laplace transformation is used to solve a linearized groundwater flow equation.

Ram and Chauhan (1987) and Singh et al (1991) have studied water table fluctuations due to transient recharge in a 1-D aquifer system with inclined base. In 1994, Ramanal et al has been extended this work in a 2-D finite aquifer system. Solution of this problem is obtained by using the methods of Fourier sine transform and eigenvalue-eigenfunction expansion.

Rao and Sarma (1984) have studied a 1-D problem of water table fluctuation in response to constant recharge from a strip basin to an aquifer which is bounded by stream and impermeable boundaries. An analytical solution to describe the dynamic position of the water table is obtained by using the extended finite Fourier transform. Then in 1996, the work of Rao and Sarma has been extended for a 2-D flow problem in which the aquifer is receiving time varying recharge from a rectangular area.

Manglik and Rai (1998) have presented an analytical solution to describe water table fluctuation due to time varying recharge from a single rectangular basin. However, recharging and pumping are to incorporate these two processes into modeling.

In this presented work, a numerical solution of a 1-D linearised Boussinesq equation is developed to describe water table fluctuation in the presence of time varying recharge from recharge basins for an one canal. Solution of this problem is obtained by using finite difference method.

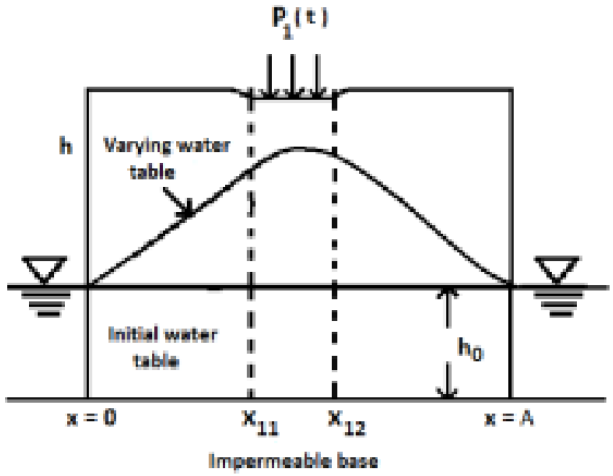


Figure 1: Vertical cross section of the 1-D flow system

2. Mathematical Formulation and Solution

A schematic diagram of a vertical cross section of the 1-D flow system is shown in Figure 1, where $P_i(t)$ is the rate of recharge for the i^{th} basin but here we take only one basin or canal and $x_{i2} - x_{i1}$ is the width of the i^{th} strip basin also h_0 is a initial water table height. The coordinate axes are taken with the origin at the boundary between the aquifer and the stream. To simplify the mathematical treatment of the problem, we assume that the aquifer is homogeneous, isotropic and resting on an impermeable horizontal base. Irrigation is applied in many parts of arid and semiarid regions to increase the production of food grains. In this process seepage from canal beds, irrigated field and other sources leads to the growth of water table near to the ground surface. The groundwater movement in the aquifer system as shown in figure 1 can be described by the following linearized Boussinesq equation [3],

$$\frac{\partial^2 H}{\partial x^2} + 2\frac{P(x,t)}{K} = \frac{1}{a}\frac{\partial H}{\partial t} \tag{1}$$

subject to the initial condition,

$$H(x,0) = 15x - \frac{x^2}{100} \tag{2}$$

and, boundary conditions,

$$H(0,t) = 0 \tag{3}$$

$$H(1500,t) = 0 \tag{4}$$

where $H = h^2 - h_0^2$, $a = \frac{K\bar{h}}{S_y}$, h is the variable water table height, h_o is the initial water table height, K is the hydraulic conductivity, S_y is the specific yield, $\bar{h}$ is the weighted mean of the depth of saturation [10], t is the time of observation, x is the distance measured from the origin of coordinates, and P(x,t) is the recharge rate defined as,

$$P(x,t) = \sum_{i=1}^{N} P_i(t) \quad for \;\; x_{i1} \leqslant x \leqslant x_{i2} \tag{5}$$

Here N is the total number of strip basins or canals (here we consider N=1) and $P_i(t)$ is the rate of recharge for i^{th} basin and $x_{i2} - x_{i1}$ is the width of the i^{th} strip basins or canals. Any number of strip basin of different width located anywhere across the x-axis can be considered.

The time varying recharge rate $P_i(t)$ is approximated by using the linear elements of different lengths and slopes in the following form,

$$P_i(t) = \begin{cases} r_{ij}t + c_{ij} & t_j \leq t \leq t_{j+1} \quad [\text{j= 1,2,...,k-1}] \\ r_{ik}t + c_{ik} & t \geq t_k \qquad\qquad [j = k] \end{cases} \tag{6}$$

where r_{ij} and c_{ij} are the slope and intercept of the jth line element of the recharge rate for ith strip basin, and k is the number of elements.

We solve the linearized Boussinesq equation (1), by using finite difference method.

We use forward difference to the time derivative and central difference to the space derivative approximation, define as follows,
i) Forward difference approximation.

$$\left(\frac{\partial T}{\partial t}\right)_i^n = \frac{T_i^{n+1} - T_i^n}{\Delta t} \tag{7}$$

ii) Central difference approximation.

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_i^n = \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{(\Delta x)^2} \tag{8}$$

By using (7) and (8), we discretised above linearized Boussinesq equation (1), as follows,

$$\frac{H_{i+1}^n - 2H_i^n + H_{i-1}^n}{(\Delta x)^2} + \frac{2P_i^n(x,t)}{K} = \frac{H_i^{n+1} - H_i^n}{a\Delta t}$$

That is,

$$H_i^{n+1} = H_i^n + \frac{a\Delta t}{(\Delta x)^2}[H_{i+1}^n - 2H_i^n + H_{i-1}^n] + \frac{2a\Delta t}{K}P_i^n \tag{9}$$

We now define,

$$r = \frac{a\Delta t}{(\Delta x)^2} \tag{10}$$

Then the equation (9), becomes

$$H_i^{n+1} = rH_{i-1}^n + (1-2r)H_i^n + rH_{i+1}^n + \frac{2a\Delta t}{K}P_i^n \tag{11}$$

3. Numerical Results and Discussion

Now, we demonstrate the application of finite difference schemes to the approximation of differential groundwater flow equation (11), or in order to illustrate the characteristic behaviours of the general solution (*equation* (11)), we have considered a synthetic numerical example in which an unconfined aquifer is receiving recharge from one canal and width is $20m$, as shown in Figure 1.

The numerical values of the controlling parameters are $A = 1500m$, $x_{11} = 740m$, $x_{12} = 760m$, $K = 8m/d$, $S = 0.2$, $h_0 = 20m$. The values of weighted mean of the depth of saturation, $\bar{h}$ are computed by using the method of successive approximation [2]. In this method the initial depth of saturation, h_0 is taken as a first approximate value of the weighted mean of the depth of saturation, $\bar{h}$. Using equation (11), the first approximate value of the water table height is calculated. Similarly, in the second trial, the mean of the initial depth of saturation and the first approximate value of the water table height is taken as $\bar{h}$ and the second approximate value of the water table height is calculated. This process is repeated till the value of the water table height is converges. This converged value is taken as the desire water table height.

Figure 2 illustrates the nature of time varying recharge. After the onset of irrigation the rate of recharge increases to a maximum value of $0.8m/d$ during 2 days and remains constant during the irrigation period which lasted till 14th days. Thereafter it decreases to zero during the next six days. This type of recharge is applied above canal.

Using equation (11), we computed water table profiles for different times t.

Let us choose the value of $\Delta x = 75$ and $\Delta t = 1$, we get a numerically stable solution to satisfy the stability condition, $r = \frac{a\Delta t}{(\Delta x)^2} \leq 0.5$ so that, $r = 0.14$. Using an initial and boundary conditions, we get unknown nodes at $n + 1th$

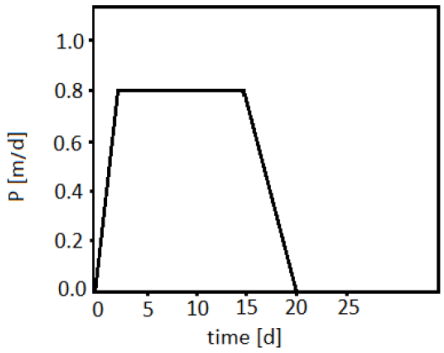


Figure 2: Time varying recharge rate

time level. Thus for $i = 0, 1, ...20$ and $n = 0$ we get

$$H_0^0 = 0, \quad H_1^0 = 1068.8, \quad H_2^0 = 2025, \quad H_3^0 = 2868.8, \quad H_4^0 = 3600,$$

$$H_5^0 = 4218.8, \quad H_6^0 = 4725, \quad H_7^0 = 5118.8, \quad H_8^0 = 5400,$$

$$H_9^0 = 5568.8, \quad H_{10}^0 = 5625, \quad H_{11}^0 = 5568.8, \quad H_{12}^0 = 5400,$$

$$H_{13}^0 = 5118.8, \quad H_{14}^0 = 4725, \quad H_{15}^0 = 4218.8, \quad H_{16}^0 = 3600,$$

$$H_{17}^0 = 2868.8, \quad H_{18}^0 = 2025, \quad H_{19}^0 = 1068.8, \quad H_{20}^0 = 0$$

The boundary conditions give $H_0^0 = H_{20}^0 = 0$. From given data, $\bar{h} = 20m$, $K = 8m/d$, $S_y = 0.20$, then $a = \frac{K\bar{h}}{S_y} = 800$. Therefore, $\frac{2a\Delta t}{K} = 200$.

Also, we find the values of $P_i^n(t)$ from Figure 2 for different days. Using above all values and the values of initial water table height in equation (11), we calculate another values of water table height.

Thus, for $i = 1$, $n = 0$ and using the numerical scheme given by equation (11), we get, $H_1^1 = 1133$.
Similarly, for $i = 2, 3, ...19$ *and* $n = 0$, we can compute $H_2^1, H_3^1, ...H_{19}^1$ which is the solution of the given initial boundary value problem for $t = \Delta t = 1$. Thus, we can use the simple algorithm described by equation (11), and compute further the values of H for subsequent time steps at all the nodes.

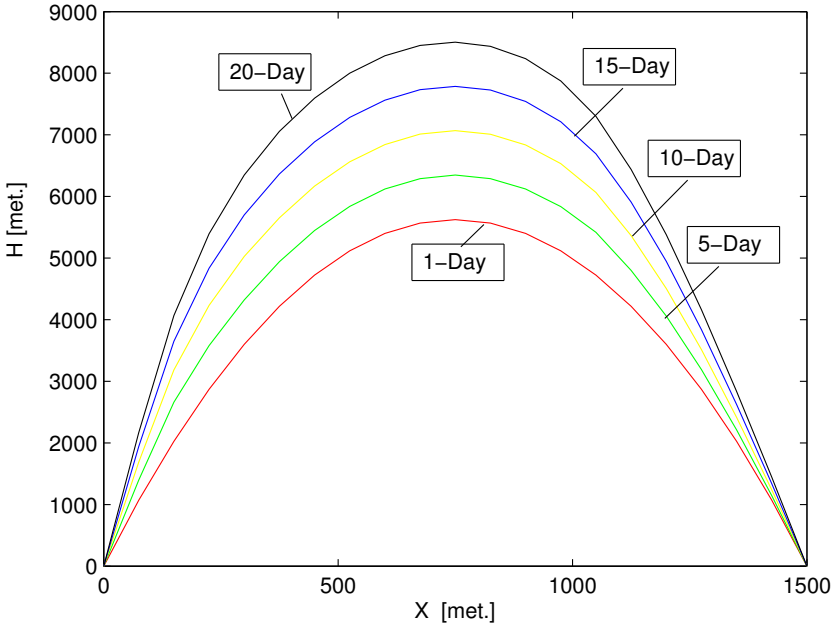


Figure 3: Water table profiles at different times

4. Conclusions

Above equation (11), is used in the computation of the water table heights for a constant rate of recharge which is equal to the initial rate of time varying recharge. From the Figure 3, it is clear that the water table continuously rises with time in the case of a constant recharge rate. On the other hand, in the case of time varying recharge, it rises with time at the beginning and after reaching to a maximum height, it starts declining. This decline is due to a decrease in the rate of recharge. So that, it is clear that the water table height simultaneously rises for different time t. During this period the growth rate is maximum below the canal.

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