A NOTE ON THE EXTENDED SONG–CHANG–PHAM’S
SOFTWARE RELIABILITY MODEL. II.

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ABSTRACT: The determination of compulsory in area of the Software Reliability
Theory components, such as confidence intervals and confidence bounds, should also
be accompanied by a serious analysis of the value of the best Hausdorff approximation
- the subject of study in the present paper.

For example we study the Hausdorff approximation of the shifted Heaviside function
$h_{t_0}(t)$ by cumulative function based on the extended Song–Chang–Pham’s model [1]
(see, also [2]).

We propose a software module within the programming environment \textit{CAS Mathematica}
for the analysis of the considered family of functions.

We give real example with dataset using the new extended Song–Chang–Pham’s
software reliability model.

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(4SHPcdf), Heaviside step–function $h_{t_0}(t)$, Hausdorff distance, upper and lower bounds

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Detailed description of all elements in the area of debugging theory may be found in the following books [4]–[6]. In the books [7]–[8], we pay particular attention to both deterministic approaches and probability models for debugging theories. A Hausdorff metric was chosen to evaluate the test data which are fitted to the sigmoid models proposed in these book. Some of the existing cumulative distributions (Gompertz–Makeham, Yamada-exponential, Yamada-Rayleigh, Yamada–Weibull, transmuted inverse exponential, transmuted Log-Logistic, Kumaraswamy–Dagum and Kumaras–wamy Quasi Lindley) are considered in the light of modern debugging and test theories. Some software reliability models, can be found in [9]–[34]. In this note we study the Hausdorff approximation of the Heaviside function \( h_{t_0}(t) \) by function based on the extended Song–Chang–Pham’s [1] cumulative function. We propose a software modules (intellectual properties) within the programming environment CAS Mathematica for the analysis. The models have been tested with real-world data.

Here we will expose from a study by Song, Chang and Pham [1]. A general mean value function \( m(t) \) of software reliability models is given by solution of the equation

\[
\frac{dm(t)}{dt} = \eta h(t)(a(t) - m(t)),
\]

where:
- \( m(t) \) is the mean value function of faults detected up to time \( t \);
- \( a(t) \) is the total number of faults in the Software at time \( t \);
- \( h(t) \) represents the fault detection rate function dependent on time \( t \)
- (assume \( \eta \)'s probability density function is \( g(\eta) \))

and \( m(t_0) = m_0 \) is the marginal condition of

\[
m(t) = \int_0^{+\infty} e^{-xH(t)} \left( m_0 + \int_{t_0}^t xa(\tau)h(\tau)e^{xH(\tau)}d\tau \right) g(x)dx,
\]

\[
H(t) = \int_0^t h(u)du.
\]

In [49] we study the Hausdorff approximation of the shifted Heaviside function \( h_{t_0}(t) \) by sigmoidal function based on the Song, Chang and Pham [1] cumulative function

\[
M(t) = N \left( 1 - \left( \frac{\beta}{\beta + \ln \frac{a + e^{-bt}}{a+1}} \right)^\alpha \right),
\]

where \( a, b, \alpha, \beta > 0, \ t > 0 \).
Definition 1. Song, Chang and Pham [1] developed the following new extended software reliability growth model:

\[ M_2(t) = N \left( 1 - \frac{\beta}{\beta + \ln \frac{a + e^{\beta t}}{a + 1}} \right)^\alpha, \]

(2)

where \(a, b, \alpha, \beta > 0, t > 0\).

Remark. The model (2) is based on the special choice:

\[ h(t) = \frac{b}{1 + ae^{-bt}} \]

in the differential equation

\[ \frac{dm(t)}{dt} = \eta h(t)(a(t) - m(t)) \]

(see, Fig. 1).

Typical confidence intervals for the model of Song, Chang and Pham are visualized on Fig. 2.

We study the Hausdorff approximation of the Heaviside function \(h_{t_0}(t)\) by function based on the extended Song–Chang–Pham’s [1] cumulative function (2).
Figure 2: Typical confidence intervals [1].

Definition 2. The shifted Heaviside step function is defined by

$$h_{t_0}(t) = \begin{cases} 
0, & \text{if } t < t_0, \\
[0, 1], & \text{if } t = t_0, \\
1, & \text{if } t > t_0 
\end{cases} \quad (3)$$

Definition 3. [35] The Hausdorff distance (the H–distance) \(\rho(f, g)\) between two interval functions \(f, g\) on \(\Omega \subseteq \mathbb{R}\), is the distance between their completed graphs \(F(f)\) and \(F(g)\) considered as closed subsets of \(\Omega \times \mathbb{R}\). More precisely,

$$\rho(f, g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B|| \},$$

wherein \(||.||\) is any norm in \(\mathbb{R}^2\), e. g. the maximum norm \(||(t, x)|| = \max\{|t|, |x|\}; hence the distance between the points \(A = (t_A, x_A), B = (t_B, x_B)\) in \(\mathbb{R}^2\) is \(||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)\).
2. MAIN RESULTS

2.1. A NOTE ON THE EXTENDED SOFTWARE RELIABILITY GROWTH MODEL (2)

The investigation of the characteristic "supersaturation" of the model (2) to the horizontal asymptote is important.

Without losing of generality we will look at the following "cumulative sigmoid":

\[
M^*_2(t) = \left(1 - \frac{\beta}{\beta + \ln \frac{a + e^{bt}}{a+1}}\right)^\alpha,
\]

with \( N = 1 \), and

\[
t_0 = \frac{1}{b} \ln \left((1 + a)e^{\beta \left(\frac{1}{1-(1/2)^{1/\alpha}} - 1\right)}\right).
\]

Evidently, for the "median" we have

\[
M^*_2(t_0) = \frac{1}{2},
\]

The one–sided Hausdorff distance \( d \) between the function \( h_{t_0}(t) \) and the cumulative function (4) satisfies the relation

\[
M^*_2(t_0 + d) = 1 - d.
\]

For example, for some conditions the following is valid

Let

\[
p_1 = -\frac{1}{2},
\]

\[
q_1 = 1 + \frac{b\alpha e^{bt_0} \left(1 - (1/2)^{\frac{1}{\alpha}}\right)^2}{2\beta(a + e^{bt_0})(1/2)^{\frac{1}{\alpha}}}.
\]

For the one–sided Hausdorff distance \( d \) between \( h_{t_0}(t) \) and the function (4) the following inequalities hold for:

\[
d_l = \frac{1}{2.1q_1} < d < \frac{\ln(2.1q_1)}{2.1q_1} = d_r.
\]

The proof follows the ideas given in [49] and will be omitted.

The model (4) for \( \beta = 0.2, \alpha = 0.8, a = 16, b = 10.1, t_0 = 0.128313 \) is visualized on Fig. 3.
Figure 3: The model (4) for $\beta = 0.2$, $\alpha = 0.8$, $a = 16$, $b = 10.1$, $t_0 = 0.128313$; H–distance $d = 0.170238$, $d_l = 0.119033$ and $d_r = 0.253344$.

Figure 4: Comparison between the models (1) (dashed) and (4) (blue).

A comparison between the models (1) (for $N = 1$) and (4) for fixed parameters $\beta = 0.2$, $\alpha = 0.8$, $a = 16$, $b = 10.1$ is visualized on Fig. 4.


2.2. NUMERICAL EXAMPLE

Example. Software Failure Data – Release #1 are presented on Fig. 5 [26].

These data derive from one major release of software products at Tandem Computers [26], [27]. 

For this data the fitted model for estimated parameters: \( N = 100; a = 77618.7; \beta = 0.000761216; \alpha = 3873812.9; b = 0.4 \) is plotted on Fig. 6.

We hope that the results will be useful for specialists in this scientific area. For other results, see [40]–[48].

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Figure 6: The fitted model $M_2(t)$ with $N = 100; a = 77618.7; \beta = 0.000761216; \alpha = 0.3873812.9; b = 0.4.$


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