International Journal of Differential Equations and Applications
Volume **19**, No. 1 (2020), pages: 167-176
ISSN (Print): 1311-2872; ISSN (Online): 1314-6084;
url: https://www.ijdea.eu

ijdea.eu

GENERALIZED MITTAG-LEFFLER STABILITY FOR IMPULSIVE CAPUTO FRACTIONAL NEURAL NETWORKS WITH BOUNDED DELAYS

Snezhana Hristova

Faculty of Mathematics and Informatics
University of Plovdiv "Paisii Hilendarski"
Plovdiv, BULGARIA

ABSTRACT: Generalized Mittag-Leffler stability of neural networks modeled by a nonlinear Caputo fractional differential equations with baounded delay and impulses is studied. We study the case whenn the lower limit of the Caputo derivative is equal to the impulsive time point, i.e. it is changeable.

Key Words: Caputo fractionalderivative, bounded delays, impulses, neural networks

Received: May 11, 2020 **Revised:** December 19, 2020
Published: December 30, 2020 **doi:** 10.12732/ijdea.v19i1.932
Academic Publications, Ltd. https://acadpubl.eu

1. INTRODUCTION

Fractional calculus has been integrated into artificial neural networks, and fractional-order neural networks are a kind of potentially applicable networks.

The application of Caputo fractional derivatives in differential equations leads to some different properties for the solutions. For example, in the case of ordinary derivative the sign of the derivative leads to a monotonicity property of the function. This is not true for the fractional case and Caputo fractional derivative. Another problem is connected with the memory property of the fractional derivatives. There are two different approaches on the interpretation of the solutions in the literature. In the first

interpretation the lower limit of the Caputo fractional derivative is the same on the whole interval of study. In the second one the lower limit of the Caputo fractional derivative is changing at each impulsive time with the idea of considering an initial value problem at each jump point. In this paper we will apply the second approach of changeable lower limit of Caputo fractional derivative to study generalized Mittag-Leffler stability of neural networks.

One of the useful methods for studying stability properties of fractional order neural networks is the Lyapunov method. The application of Lyapunov functions to the fractional differential equations requires appropriate definition of their derivatives among the studied fractional differential equation.

2. PRELIMINARY NOTATIONS AND RESULTS

We give some basic definitions of fractional calculus and introduce some useful lemmas.

In many applications in science and engineering, the fractional order q is often less than 1, so we restrict $q \in (0,1)$ everywhere in the paper.

1: *T*he Riemann-Liouville (RL) fractional integral of order $q \in (0,1)$ of $m(t)$ is given by (see for example, [4])

$$ {}_{t_0}I_t^q m(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} m(s) ds, \qquad t > t_0 $$

where $\Gamma(.)$ denotes the Gamma function.

2: *T*he Caputo fractional derivative of order $q \in (0,1)$ is defined by (see for example, [4])

$$ {}_{t_0}^{C}D_t^q m(t) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} (t-s)^{-q} m'(s) ds, \qquad t > t_0. $$

The above definitions are given for scalar real vaued functions but they can easily be generalized for vector functions, real and complex valued.

3. SOME PRELIMINARY RESULTS FOR IMPULSIVE CAPUTO FRACTIONAL DIFFERENTIAL EQUATIONS WITH DELAY.

Let the positive constant $r > 0$ be given and the points $t_i \geq 0$ be such that $0 < t_i < t_{i+1}, i = 1, 2, \ldots$.

Consider the space $PC_0 = C([-r, 0], E)$ endowed with the norm

$$||y||_{PC_0} = \sup_{t\in[-r,0]} \{||y(t)||_E : \ y \in PC_0\}$$

where E is a Banach space.

Consider the case when the lower limit of the fractional derivative is changeable at each interval without impulses

Consider the initial value problem (IVP) for a nonlinear system of impulsive fractional differential equations with state dependent delay(IFrDDE) with $q \in (0, 1)$

$$\begin{aligned} &{}^{C}_{t_i}D^q_t y(t) = f(t, y_t) \ \text{ for } t = (t_i, t_{i+1}], i = 0, 1, 2, \dots, \\ &y(t_i + 0) = I_i(y(t_i)), \quad i = 1, 2, \dots, \\ &y(t) = \phi(t) \ \text{ for } t \in [-r, 0], \end{aligned} \tag{3.1}$$

where ${}^{C}_{t_i}D^q_t$ denotes the Caputo's fractional derivative with lower bound t_i for the state $y(t)$ over the interval $(t_i, t_{i+1}]$.

Let $\mathcal{PC}_2$ be the Banach space of functions $y : [-r, T] \to E$ which are continuous on $[0, T]$ except for the points $t_i \in (0, T)$ at which $y(t_i+)$ and $y(t_i-) = y(t_i)$ exist, for $t \in (s_i, t_{i+1}]$ the Caputo fractional derivative ${}^{C}_{s_i}D^q_t y(t), i = 0, 1, \dots, k$, exists and it is endowed with the norm $||y||_{\mathcal{PC}_2} = \sup_{t\in[-r,T]}\{||y(t)||_E : y \in \mathcal{PC}_2\}$. Let $PC^q_2[0, T] = \{u \in C(\bigcup_{i=1}^{k}(s_i, t_{i+1}], E)$ such that for any $t \in (s_i, t_{i+1}]$, $\int_{s_i}^{t}(t - s)^{q-1}u(s)ds < \infty$, $i = 1, 2, \dots, k\}$.

Definition 1. The zero solution of IFrDDE (3.1) with zero initial function is called *generalized Mittag-Leffler stable* if there exist a constant $\beta \in (0, 1)$ and positive constants a, b, M, μ such that the inequality

$$\begin{aligned} ||x(t)|| \le M||\phi||^b_0 \Big(\big(\prod_{i=0}^{k-1} E_\beta(-\mu(t_{i+1} - t_i)^\beta)\big) E_\beta(-\mu(t - t_k)^\beta)\Big)^a, \\ \text{for } t \in (t_k, t_{k+1}], k = 0, 1, \dots \end{aligned} \tag{3.2}$$

holds, where $x(t) = x(t; t_0, \phi)$ is a solution of (3.1) and $E_\beta(z)$ is the Mittag-Leffler function with one parameter β.

Remark 1. Note the parameter β of the Mittag-Leffler function could be different than the fractional order q of the given IFrDDE (3.1).

We will use the Caputo fractional Dini derivative among trajectories of solutions of

IVP for the system NIFrDDE (3.1) given by:

$$
\begin{aligned}
{}^{c}_{(3.1)}D^{q}_{+}V(t,\psi(0),\psi;t_0,\phi) &= \limsup_{h\to 0^+}\frac{1}{h^q}\Big\{V(t,\psi(0)) - V(t_0,\phi(0)) \\
&- \sum_{r=1}^{[\frac{t-t_0}{h}]}(-1)^{r+1}\binom{q}{r}\Big[V(t-rh,\psi(0)-h^q f(t,\psi)) - V(t_0,\phi(0))\Big]\Big\} \\
&\qquad \text{for } t\in(t_k,t_{k+1}),\ k=0,1,\dots,
\end{aligned}
\tag{3.3}
$$

where $\psi \in C([-r,0],E)$ is an arbitrary function, (t_0,ϕ) is the initial data of the IVP for NIFrDDE (3.1).

Remark 2. Let $\psi \in C([-\tau,0],E)$ be given and $V(t,x) = m(t)\sum_{i=1}^{n} x_i^2$ where $m \in C(\mathbb{R}_+,\mathbb{R}_+)$. Then

$$
{}^{c}_{(3.1)}D^{q}_{+}V(t,\psi(0),\psi;t_0,\phi) \le 2m(t)\sum_{i=1}^{n}\psi_i(0)\ f_i(t,\psi_0) + {}^{RL}_{t_k}D^{q}\Big(m(t)\Big)\sum_{i=1}^{n}(\psi_i(0))^2. \tag{3.4}
$$

We will use the following result which is a partial case of Theorem 4.1 [3] for instantaneous impulses:

Theorem 1. *Let*

1. *The function $f \in C(\mathbb{R}_+\times\mathbb{R}^n,\mathbb{R}^n)$, $f(t,0)\equiv 0$.*
2. *There exists a function $V \in \Lambda([-r,\infty),\Delta)$ such that*

(i) $\alpha_1||x||^a \le V(t,x) \le \alpha_2||x||^{ab}$ *for* $t\ge 0$, $x\in\mathbb{R}^n$, *where* α_1,α_2,a,b *are positive numbers;*

(ii) for any $i=0,1,2,\dots$ *and any function* $\psi\in E$ *such that for a point* $t\in(t_i,t_{i+1}]$ *the inequality* $V(t,\psi(0)) \ge \sup_{s\in[-r,0]}V(t+s,\psi(s))$ *implies*

$$
{}^{c}_{(3.1)}D^{q}_{+}V(t,\psi(0),\psi;t_0,\phi) \le -\alpha_3||\psi||_0^{ab} \tag{3.5}
$$

holds;

(iii) for any $k=0,1,2,\dots$ *and* $y\in\mathbb{R}^n$ *the inequality*

$$
V(t,I_k(y)) \le \alpha_4||y||^a \quad \text{for } t\in(t_k,t_{k+1}],
$$

holds where α_4 *is a positive constant such that* $\alpha_4\le\alpha_1$.

Then the zero solution of the IVP for IFrDDE (3.1) with zero initial function is generalized Mittag-Leffler stable.

Remark 3. From [3] if conditions of Theorem 1 are satisfied then the zero solution of the IVP for IFrDDE (3.1) is generalized Mittag-Leffler stable with $\mu = \frac{\alpha_3}{\alpha_2}$, $\beta = q$, $a = \frac{1}{\alpha_1}$.

4. STATEMENT OF THE PROBLEM

In this paper we will assume the increasing sequence of points $\{t_i\}_{i=1}^{\infty}$ is given such that $0 < t_i < t_{i+1}$, $i = 1, 2, \ldots$, and $\lim_{k\to\infty} t_k = \infty$.

Let $t_0 \in \mathbb{R}_+$ be a given arbitrary point. Without loss of generality we will assume that $0 \le t_0 < t_1$.

Consider the general model of Hopfield's graded response neural networks with impulses and delays(IDNN)

$$\begin{aligned} &{}^{C}_{t_k}D^q_t x_i(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)g_j((x_j)_t) + I_i(t) \\ &\quad \text{for } t \in (t_k, t_{k+1}], \quad i = 1, 2, \ldots n, \\ &x_i(t_k + 0) = I_{k,i}(x_i(t_k - 0)) \qquad \text{for } \ k = 1, 2, \ldots \\ &x_i(t_0 + s) = \phi_i(s) \quad s \in [-r, 0], \end{aligned} \tag{4.1}$$

where n represents the number of neurons in the network, $x_i(t)$ is the pseudostate variable denoting the average membrane potential of the i-th neuron at time t, $c_i(t) > 0, i = 1, 2, \ldots, n$, is the self-regulating parameter of the i-th unit, $\tau : \mathbb{R}_+ \to [-r, \infty)$, $t - r \le \tau(t) \le t$, is the time delay, $a_{ij}(t)$, $i, j = 1, 2, \ldots, n$, correspond to the synaptic connection strength of the i-th neuron to the j-th neuron at time t , $f_j(x_j(t))$, $g_j((x_j)_t)$ denote the activation functions of the neurons at time t and the delay time on the interval $[t - \tau, t]$, and they respectively, represent the response of the j-th neuron to its membrane potential, $f(x) = (f_1(x_1), f_2(x_2), \ldots, f_n(x_n))$, $g(x) = (g_1(x_1), g_2(x_2), \ldots, g_n(x_n))$ and $I = (I_1, I_2, \ldots, I_n)$ is an external bias vector, the points $t_k, k = 1, 2, \ldots$, time of the impulsive action, the functions $I_{k,i}(u)$, $k = 1, 2, \ldots$ are the functions giving the amount of the jump of the i-th neuron.

Definition 2. A vector $x^* \in \mathbb{R}^n$, $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ is an equilibrium point of IDNN (4.1), iff the equalities

$$0 = -c_i(t)x_i^* + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j^*) + \sum_{j=1}^{n} b_{ij}(t)g_j(x_j^*) + I_i(t) \text{ for } t \in (t_k, t_{k+1}], \ i = 1, 2, \ldots, n \tag{4.2}$$

and

$$x_i^* = I_{k,i}(x_i^*) \ \text{ for } k = 1, 2, \dots, \ i = 1, 2, \dots, n \tag{4.3}$$

hold.

5. GENERALIZED MITTAG-LEFFLER STABILITY

Using the idea of the generalized Mittag-Leffler stability of the zero solution of IFrDDE (3.1) with zero initial function defined by Definition 1 we will define a similar stability property for IDNN (4.1):

Definition 3. The equilibrium point of IDNN (4.1) is called *generalized Mittag-Leffler stable* if there exist a constant $\beta \in (0, 1)$ and positive constants a, b, M, μ such that the inequality

$$\begin{aligned} ||x(t) - x^*|| \leq M||\phi - x^*||_0^b \Big(\big(\prod_{i=0}^{k-1} E_\beta(-\mu(t_{i+1} - t_i)^\beta) \big) E_\beta(-\mu(t - t_k)^\beta) \Big)^a, \\ \text{for} \quad t \in (t_k, t_{k+1}], k = 0, 1, \dots \end{aligned} \tag{5.1}$$

holds, where $x(t) = x(t; t_0, \phi)$ is a solution of IDNN (4.1) and $E_\beta(z)$ is the Mittag-Leffler function with one parameter β.

We assume the following:

Assumption A1. Let the IDNN (4.1) have an equilibrium vector $x^* \in \mathbb{R}^n$.

If assumption A1 is satisfied then we can shift the equilibrium point x^* of system (4.1) to the origin. The transformation $y(t) = x(t) - x^*$ is used to put system (4.1) in the following form:

$$\begin{aligned} &{}_{t_k}^{C} D_t^q y_i(t) = -c_i(t) y_i(t) + \sum_{j=1}^{n} a_{ij}(t) F_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}(t) G_j((y_j)_t) \\ &\qquad \text{for } t \in \cup_{k=0}^{\infty}(t_k, t_{k+1}], \ i = 1, 2, \dots n \\ &y_i(t) = J_{k,i}(y(t_k - 0)) \qquad \text{for} \quad k = 1, 2, \dots \\ &y_i(t_0 + s) = \Phi_i(s), \quad s \in [-r, 0], \end{aligned} \tag{5.2}$$

where $F_j(u) = f_j(u + x_j^*) - f_j(x_j^*), j = 1, 2, \dots, n$, $G_j(u) = g_j(u + x_j^*) - g_j(x_j^*), j = 1, 2, \dots, n$ and $J_{k,i}(u) = I_{k,i}(u + x_i^*) - I_{k,i}(x_i^*), \ i = 1, 2, \dots, n, \ k = 1, 2, \dots,$, $\Phi_i(s) = \phi_i(s) - x_i^*, \quad s \in [-r, 0]$.

Note if the point $x^* \in \mathbb{R}^n$ is an equilibrium of IDNN (4.1) then the point $y^* = 0$ is an equilibrium of IDNN (5.2). This allows us to study the stability properties of the zero equilibrium of of IDNN (5.2).

We will introduce the following assumptions:

Assumption A2. The neuron activation functions are locally Lipschitz, i.e. there exist positive numbers $L_i,\ i = 1, 2, \dots, n$ such that $|f_i(u) - f_i(v)| \le L_i|u - v|,\ i = 1, 2, \dots, n$ for $u, v \in \mathbb{R} :\ |u - x_i^*| \le \lambda, |v - x_i^*| \le \lambda$ where the point $x^* \in \mathbb{R}^n$ is the equilibrium from assumption A1.

Assumption A3. There exists a non- decreasing function $m \in C(\mathbb{R}_+, [A, B])$, $A, B > 0$, such that the inequality

$$\begin{aligned}&\Big(2m(t)\min_i\ c_i(t) - {}_{t_k}^{RL}D^q\Big(m(t)\Big)\Big)\\ &- m(t)\Big[\sum_{j=1}^{n}\Big(L_j\max_i|a_{ij}(t)| + K_j\max_i|b_{ij}(t)|\Big) + L_j\sum_{i=1}^{n}\max_j|a_{ij}(t)|\\ &\quad + K_j\sum_{i=1}^{n}\max_j|b_{ij}(t)|\Big] \ge C \quad \text{for } t \ge 0\end{aligned} \tag{5.3}$$

holds.

Assumption A4. There exists a positive number $\lambda \le 1$ such that for $k = 1, 2, \dots$ and $x \in \mathbb{R}^n$ the inequalities

$$\sum_{i=1}^{n}\Big(I_{k,i}(x_i + x_i^*) - I_{k,i}(x_i^*)\Big)^2 \le \lambda||x - x^*||^2,\quad k = 1, 2, \dots$$

hold where x^* is the equilibrium point from assumption A1.

Remark 4. If assumption A2 is fulfilled then the function F in IDNN (5.2) satisfies the inequalities $|F_j(u)| \le L_j|u|,\ j = 1, 2, \dots, n$ for any $u :\ |u| \le \lambda$.

Remark 5. If assumption A4 is satisfied then $\sum_{i=1}^{n}\Big(J_{k,i}(x_i)\Big)^2 \le \lambda||x||^2,\quad k = 1, 2, \dots$.

Theorem 2. *Let assumptions A1-A4 be satisfied.*

Then the equilibrium point x^* of IDNN (4.1) is generalized Mittag-Leffler stable.

Proof. Consider the Lyapunov function $V(t, x) = m(t)x^Tx$. Thus the condition (i) of Theorem 1 is satisfied with $a = 2,\ b = 1,\ \alpha_1 = A, \alpha_2 = B$.

Let the function $\psi \in C([-\tau, 0], E)$ be such that for a point $t \in (t_i, t_{i+1}]$ the inequality $m(t)\sum_{i=1}^{n}(\psi(0))^2 \ge \sup_{s\in[-r,0]} m(t+s)\sum_{i=1}^{n}(\psi(s))^2 \ge m(t)\sup_{s\in[-r,0]}\sum_{i=1}^{n}(\psi(s))^2$

hods. Then according to Remark 2 we get

$$
\begin{aligned}
&{}^{c}_{(3.1)}D^{q}_{+}V(t,\psi(0),\psi;t_0,\phi)\\
&\leq -2m(t)\sum_{i=1}^{n}(\psi_i(0))^2c_i(t)+2m(t)\sum_{i=1}^{n}\psi_i(0)\sum_{j=1}^{n}a_{ij}(t)F_j(\psi_j(0))\\
&+2m(t)\sum_{i=1}^{n}\psi_i(0)\sum_{j=1}^{n}b_{ij}(t)G_j((\psi_j)_0)+{}^{RL}_{t_k}D^{q}\Big(m(t)\Big)\sum_{i=1}^{n}(\psi_i(0))^2\\
&\leq -\sum_{i=1}^{n}(\psi_i(0))^2\Big(2m(t)c_i(t)-{}^{RL}_{t_k}D^{q}\Big(m(t)\Big)\Big)\\
&+2m(t)\sum_{i=1}^{n}|\psi_i(0)|\sum_{j=1}^{n}|a_{ij}(t)|L_j|\psi_j(0)|+2m(t)\sum_{i=1}^{n}|\psi_i(0)|\sum_{j=1}^{n}|b_{ij}(t)|K_j|(\psi_j)_0|\\
&\leq -\Big(2m(t)\min_t\ c_i(t)-{}^{RL}_{t_k}D^{q}\Big(m(t)\Big)\Big)||\psi(0)||^2\\
&+m(t)\sum_{i=1}^{n}(\psi_i(0))^2\sum_{j=1}^{n}\Big(|a_{ij}(t)|L_j+|b_{ij}(t)|K_j\Big)\\
&+m(t)\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(|a_{ij}(t)|L_j+|b_{ij}(t)|K_j\Big)((\psi_j)_0)^2\\
&\leq -\Big(2m(t)\min_i\ c_i(t)-{}^{RL}_{t_k}D^{q}\Big(m(t)\Big)\Big)||\psi(0)||^2\\
&+m(t)\Big[\sum_{j=1}^{n}\Big(L_j\max_i|a_{ij}(t)|+K_j\max_i|b_{ij}(t)|\Big)\\
&+L_j\sum_{i=1}^{n}\max_j|a_{ij}(t)|+K_j\sum_{i=1}^{n}\max_j|b_{ij}(t)|\Big]||\psi(0)||^2\\
&\leq -C(||\psi||_0)^2.
\end{aligned}
\tag{5.4}
$$

For any $k=0,1,2,\dots$ and $y\in\mathbb{R}^n$ according to condition A4 and Remark 5 and assumption A3 we get the inequality

$$
\begin{aligned}
&V(t,I_k(y))=m(t)\sum_{i=1}^{n}(J_{k,i}(y_i))^2\leq m(t)\sum_{i=1}^{n}M_{k,i}(y_i)^2\leq m(t)\max_i M_{k,i}\sum_{i=1}^{n}(y_i)^2\\
&\leq m(t)\max_i M_{k,i}||y||^2\leq\alpha_4||y||^2\quad\text{for } t\in(t_k,t_{k+1}].
\end{aligned}
\tag{5.5}
$$

where $\alpha_4=B\lambda<\alpha_2=B$. Thus, the condition (iii) of Theorem 1 is satisfied.

From Theorem 1 the zero solution of IDNN (5.2) with zero initial function is generalized Mittag-Leffler stable, i.e. the equilibrium point x^* of IDNN (4.1) is generalized Mittag-Leffler stable. □

5.1. PARTIAL CASE

Consider the case of the model of Hopfield's graded response neural networks with impulses and bounded delays with constant self-regulating parameters $c_i(t) \equiv c > 0$ and constant synaptic connection strengths $a_{ij}(t) \equiv a_{ij}, a_{ij}(t) \equiv a_{ij}, \quad i, j = 1, 2, \ldots, n$. Then the assumption A3 is reduced to

Assumption A3*. There exists a positive number C, such that the inequality

$$2 \min_i c_i - \Big[\sum_{j=1}^{n} \Big(L_j \max_i |a_{ij}| + K_j \max_i |b_{ij}| \Big) + L_j \sum_{i=1}^{n} \max_j |a_{ij}| + K_j \sum_{i=1}^{n} \max_j |b_{ij}| \Big] \geq C. \tag{5.6}$$

holds.

In this case in the proof we apply the quadratic Lyapunov function. Also, in this paprtial case we could apply the Caputo fractional derivative of the Lyapunov function instead of the Caputo fractional Dini derivative.

6. CONCLUSIONS

We study the general model of Hopfield's graded response neural networks with impulses and bounded delays in the general case of variable in time self-regulating parameters and the synaptic connection strengths. In applications, when Hopfield-type neural networks are used for associative memories, the equilibrium states of the network serve as stored patterns and their stability implies that the stored patterns can be retrieved in the presence of perturbations of the network which can be non-instantaneous impulsive state displacements. When Hopfield networks are used for the solution of optimization problems, the equilibrium states denote the possible optimal solutions and their stability will guarantee the convergence to the solutions beginning from an appropriate initial state. If the equilibrium is unique and the stability is a global asymptotic one, then the initial starting state can be an arbitrary one. The stability of a network is one of the desirable characteristics for the implementation of a network.

In this paper we obtain sufficient conditions for generalized Mittag-Leffler stability of the equilibrium point x^* of IDNN (4.1) with $\mu = \frac{C}{B}$, $\beta = q$, $a = \frac{1}{A}$, $M = \frac{B}{A}$, i.e. the sufficient condition for any solution to satisfy the inequality

$$||x(t) - x^*|| \leq \frac{B}{A} ||\phi - x^*||_0 \Big(\big(\prod_{i=0}^{k-1} E_\beta(-\frac{C}{B}(t_{i+1} - t_i)^q) \big) E_\beta(-\frac{C}{B}(t - t_k)^q) \Big)^{\frac{1}{A}}, \tag{6.1}$$

$$\text{for} \quad t \in (t_k, t_{k+1}], k = 0, 1, \ldots.$$

ACKNOWLEDGMENTS

The research is supported by the Bulgarian National Science Fund under Project KP-06-N32/7 and Project MU19FMI009.

REFERENCES

[1] S. Abbas, M. Benchohra, Stability results for fractional differential equations with state-dependent delay and not instantaneous impulses, *Math. Slovaca*, **67**, No. 4 (2017), 875-894.

[2] Ravi Agarwal, S. Hristova, D. O'Regan, Non-instantaneous impulses in Caputo fractional differential equations, *Fract. Calc. Appl. Anal.*, **20**, No. 3 (2017), 595-622.

[3] Ravi Agarwal, S. Hristova, D. O'Regan, Mittag-Leffler stability for non-instantaneous impulsive Caputo fractional defferential equations with delays, *Math. Slovaca*, **69**, No. 3 (2019), 583-598.

[4] Sh. Das, *Functional Fractional Calculus*, Springer-Verlag Berlin Heidelberg, 2011.

[5] J. Wang, Y. Zhou, M. Feckan, Nonlinear impulsive problems for fractional differential equations and Ulam stability, *Comput. Math. Appl.*, **64** (2012), 3389-3405.

[6] Wu H.Q., Zhang X.X., Xue S.H.,Wang L.F.,Wang Y., LMI conditions to global Mittag-Leffler stability of fractional-order neural networks with impulses, *Neurocomputing*, **193** (2016), 148-154.

[7] I. Stamova, Global Mittag-Leffler stability and synchronization of impulsive fractional-order neural networks with time varying delays, *Nonlinear Dynamics*, **77** (2014), 1251-1260.

[8] I. Stamova, G. Stamov, Stability analysis of impulsive functional systems of fractional order, *Commun. Nonlinear Sci. Numer. Simulat.*, **19** (2014), 702-709.

[9] H.M. Srivastava, S. Abbas, S. Tyagi, D. Lassoued, Global exponential stability of fractional-order impulsive neural network with time-varying and distributed delay, *Math. Meth. Appl. Sci.* (2018), 1-10.

[10] S. K. Choi, N. Koo, The monotonic property and stability of solutions of fractional differential equations, *Nonlinear Analysis*, **74** (2011), 6530-6536.