International Journal of Differential Equations and Applications

Volume 9 No. 3 2004, 173-191

WAVELET-GALERKIN SOLUTION OF BBM EQUATION

A.A. Ashour

Department of Mathematical and Physical Sciences
Faculty of Engineering
Mansoura University
Mansoura, 35516, EGYPT
e-mail: ashour123@mans.edu.eg

Abstract: In this paper a new method for solving a periodic nonhomogeneous nonlinear Benjamin, Bona and Mahony (BBM) equation using wavelet basis is developed. This method is used to solve some illustrative examples and the obtained numerical solutions are compared with the corresponding analytic solutions.

AMS Subject Classification: 65T60, 35K55
Key Words: wavelet-Galerkin method, connection coefficients, multiresolution analysis, BBM equation, nonlinear partial differential equations

1. Introduction

In this work, we consider the Benjamin, Bona and Mahony (BBM) equation

Received: October 11, 2004

$$\frac{\partial u}{\partial t} + \alpha \frac{\partial u}{\partial x} + \beta\, u \frac{\partial u}{\partial x} - \gamma \frac{\partial^3 u}{\partial x^2 \partial t} = f(x\ ,\ t)\ , \tag{1.1}$$

on the space-time domain $Q \equiv [0\ ,1] \times (0\ ,T]$ subject to the boundary conditions

$$u(0\ ,\ t) = u(1,\ t) = u_{b_1}, \quad \forall t \in (0\ ,\ T] \tag{1.2}$$

and the initial condition

$$u(x\ ,\ 0) = u_0(x), \quad \text{in } \Omega\,. \tag{1.3}$$

Here α, β and γ are constants, $f(x,\, t)$ is a Lipschitz continuous real function and the initial function $u_0(x)$ is a bounded continuous function. This equation was first put forward by Peregrine to describe the development of an undular bore and wave motion to the same order of approximation as the KdV equation. Since the BBM equation has the same formal justification as the KdV equation, it is also a well applicable model. Though difference between KdV and BBM equations is only reflected in the emergence of u_{xxx} or u_{xxt}, they have some real differences: KdV as a completely integrable model has N solitary solutions, but if there is the corresponding result for the BBM equation is unknown, since the interaction of two or three solitons is inelastic, see [11].

In this work, we suggest a new method based on the wavelet-Galerkin method to solve this problem. The proposed technique is programmed using *Matlab* and used to solve some examples of the desired problem. The obtained numerical results are compared with the exact solutions of the desired examples.

The recognition and the study of several spatial phenomena strongly depends on the resolution at which the analysis is carried out. The availability of an efficient way to represent the same phenomena at different resolutions is therefore very important (from both a mathematical and a computational point of view). Wavelets, families of bases functions generated by dilation and translation of a mother function, provide a good balance of localization in space and localization in the frequency domain, unlike the Fourier transform (where the localization in space is lost) and spline (where the localization in frequency is lost). Wavelets can be used in a multiresolution analysis, where the signal and the difference from the previous resolution level are orthogonal, to provide an efficient tool for investigating resolution dependent phenomena, see [19, 20].

Orthonormal wavelets provide explicit unconditional bases for many of the important function spaces in harmonic analysis, partial differential equations (PDE), and Banach space theory [3, 12]. A reason for the popularity of wavelet is its effectiveness in representation of transient signals, since most of natural and human-made signals are transient in nature. Daubechies' wavelets constructed by I. Daubechies [9, 10] have gained great interest in the numerical solution of integral equations, differential equations and integro-differential equations (see for example [1, 2, 5, 8, 14, 15]). Throughout this work we use the notation $(. \, , \, .)$ to denote the standard inner product in $L_2(\Omega)$ which defined as

$$(f(x) \; , \; g(x)) = \int\limits_{\Omega} f(x) \; g(x) \; dx \; .$$

The paper is organized as follows. Section 2 presents some basic concepts of wavelet bases and multiresolution analysis. Section 3 describes our algorithm for solving the periodic nonhomogenous BBM equation. In Section 4, we use our technique to solve some numerical examples and the results are compared with the corresponding analytic solutions.

2. Background and Notation

Ingrid Daubechies constructed a class of compactly supported wavelets. The construction of wavelets is related with the construction of a scaling function $\varphi(x)$ such that for fixed $J \in Z$ the function

$$\varphi_{J\,,\,k}(x) \;=\; 2^{J/2}\,\varphi(2^J x \;-\; k)\;, \quad J\; \geq 0,\; k\;\in\; Z\,, \tag{2.1}$$

forms a Riesz basis for a subspace V_J of the space of square integrable functions $L_2(R)$. Moreover, the spaces V_J constitute a multiresolution analysis for $L_2(R)$, i.e. for $J \in Z$,

$$V_{J-1} \;\subset\; V_J \;, \tag{2.2}$$

with

$$\bigcap_{J \in Z} V_J = \{0\} , \tag{2.3}$$

and

$$\overline{\bigcup_{J \in Z} V_J} = L_2(R) , \tag{2.4}$$

The wavelet spaces W_J are the orthogonal complements of V_{J+1} in V_J. One defines the wavelet function such that the functions

$$\psi_{J\ ,\ k}(x) = 2^{J/2}\ \psi(2^J x - k) , \qquad J \geq 0 ,\ k \in Z \tag{2.5}$$

forms an orthonormal basis for a subspace W_J. Since both V_J and W_J are contained in V_{J+1} the scaling function $\varphi(x)$ must satisfy the dilation equation

$$\varphi(x) = \sqrt{2} \sum_{k=0}^{D-1} a_k\ \varphi(2x - k) , \tag{2.6}$$

where D is an even integer greater than two and is called Daubechies number. The set $\{a_k\}_{k=0}^{D-1} \in l^2$ are called the filter coefficients. The scaling function $\varphi(x)$ has a compact support in the interval $[0\ ,\ D-1]$. Correspondingly, the wavelet $\psi(x)$ satisfies

$$\psi(x) = \sqrt{2} \sum_{k=0}^{D-1} (-1)^k\ a_{D-1-k}\ \varphi(2x - k) , \tag{2.7}$$

The wavelet function $\psi(x)$ has a compact support in the interval $[1 - \frac{D}{2}\ ,\ \frac{D}{2}]$.

The scaling and wavelet functions have many properties, such as:

$$(\varphi(x)\ ,\ \varphi(x)) = 1 , \tag{2.8}$$

$$(\varphi(x)\ ,\ 1) = 1 , \tag{2.9}$$

$$\sum_{k=0}^{D-1} \varphi(k) = 1 \ , \tag{2.10}$$

$$(\psi(x) \ , \ \psi(x)) = 1 \ , \tag{2.11}$$

$$(\psi(x-k) \ , \ \psi(x-l)) = \delta_{k\ ,\ l} \ , \tag{2.12}$$

$$(\varphi(x) \ , \ \psi(x)) = 0 \ , \tag{2.13}$$

$$(x^p \ , \ \psi(x-k)) = 0 \ , \quad p = 0 \ , \ 1 \ , \ ..., \ \frac{D}{2} - 1 \ , \quad l \in Z \, , \tag{2.14}$$

where $\delta_{k,l}$ is the Kronecker delta defined by

$$\delta_{k\ ,\ l} = \begin{cases} 1 & \text{if } k = l \\ 0 & \text{if } k \neq l \end{cases}$$

The wavelets discussed above are defined on the whole real line R. These wavelets can be periodized to obtain periodic wavelets [16, 17]. The periodized scaling and wavelet basis are applicable when the domain of the problem is infinite or when the boundary conditions are periodic. Assuming that the interval $\Omega = [0\ , 1]$, then the 1-periodic scaling and wavelet functions are defined as:

$$\tilde{\varphi}_{J,\ k}(x) = \sum_{l \in Z} \varphi_{J,\ k}(x+l) \ , \tag{2.15}$$

$$\tilde{\psi}_{J,\ k}(x) = \sum_{l \in Z} \psi_{J,\ k}(x+l). \tag{2.16}$$

From these relations we obtain the following relations

$$\tilde{\varphi}_{J,\,k}(x) = \begin{cases} 2^{J/2}\varphi(2^J x - k)\ , & x \in \Omega \cap x \in I_{J,\,k}\,, \\ 2^{J/2}\varphi(2^J(x+1) - k)\ , & x \in \Omega \cap x \notin I_{J,\,k}\,, \end{cases} \tag{2.17}$$

$$\tilde{\psi}_{J,\,k}(x) = \begin{cases} 2^{J/2}\psi(2^J x - k)\ , & x \in \Omega \cap x \in I_{J,\,k}\,, \\ 2^{J/2}\psi(2^J(x+1) - k)\ , & x \in \Omega \cap x \notin I_{J,\,k}\,, \end{cases} \tag{2.18}$$

where $I_{J,\,k} = \text{supp}\ (\varphi_{J,\,k}(x)) = [k/2^J,\ (k+D-1)/2^J]$. It is assumed that $J \geq J_0$ where J_0 is the smallest integer such that $2^{J_0} \geq D - 1$. For every $J \geq J_0$, the set of scaling functions $\{\tilde{\varphi}_{J,\,k}\}_{k=0}^{2^J-1}$ forms an orthonormal basis of a subspace of $L_2(\Omega)$ which we denote V_J. As $\tilde{\varphi}_{J,\,k} = \tilde{\varphi}_{J,\,\bar{k}}$ if $k \equiv \bar{k}$ mod (2^J). So our attention is restricted to $0 \leq k < 2^J$. Similarly for $\tilde{\psi}'s$. So, a periodic function $f(x) \in V_J$ can be expanded in the following two ways:

The pure periodic scaling function expansion

$$f(x) = \sum_{l=0}^{2^J-1} c_{J,\,l}\ \tilde{\varphi}_{J,\,l}(x)\ , \quad x \in \Omega\,, \tag{2.19}$$

and the periodic wavelet expansion

$$f(x) = \sum_{l=0}^{2^{J_0}-1} c_{J_0,\,l}\ \tilde{\varphi}_{J_0,\,l}(x) + \sum_{j=J_0}^{J-1}\sum_{l=0}^{2^j-1} d_{j,\,l}\ \tilde{\psi}_{j,\,l}(x)\ , \quad x \in \Omega\,, \tag{2.20}$$

where the coefficients $c_{J_0,\,l}$ and $d_{j,\,l}$ are defined as

$$c_{J_0,\,l} = (f(x)\ ,\ \tilde{\varphi}_{J_0,\,l}(x))\,, \tag{2.21}$$

$$d_{j,\,l} = (f(x)\ ,\ \tilde{\psi}_{j,\,l}(x))\,. \tag{2.22}$$

For more details the reader is referred to [13, 17].

3. Wavelet-Galerkin Solution of a Periodic BBM Equation

We consider the nonhomogeneous periodic BBM problem which takes the following mathematical model:

Problem P. Find $u : \Omega \times I \to R$ such that

$$\frac{\partial u}{\partial t} + \alpha \, \frac{\partial u}{\partial x} + \beta \, u \, \frac{\partial u}{\partial x} - \gamma \, \frac{\partial^3 u}{\partial x^2 \partial t} = f(x,t), \quad \text{in } \Omega \times I \, , \tag{3.1}$$

$$u(0,t) = u(1,t) = u_b, \tag{3.2}$$

$$u(x, \ 0) = u_0(x), \quad \text{in } \Omega \, . \tag{3.3}$$

Here, $I = (0, \ T)$, $T < \infty$, $\Omega = [0, \ 1]$ with boundary $\partial\Omega = \{0, \ 1\}$ and the functions $u(x)$, $u_0(x)$ and $f(x,t)$ are continuous periodic functions in x of period 1.

The wavelet-Galerkin method entails representing the solution function $u(x,t)$ and the source function $f(x,t)$ as expansions of scaling functions at a particular resolution J as

$$u_J(x,t) = \sum_{k=0}^{2^J-1} c_{u_{J,\ k}}(t) \ \tilde{\varphi}_{J,\ k}(x) \ , \tag{3.4}$$

$$f_J(x,t) = \sum_{k=0}^{2^J-1} c_{f_{J,\ k}}(t) \ \tilde{\varphi}_{J,\ k}(x) \ , \tag{3.5}$$

where,

$$c_{u_{J,\ k}}(t) = (u(x,t) \ , \ \tilde{\varphi}_{J,\ k}(x)) \ , \tag{3.6}$$

$$c_{f_{J,\ k}}(t) = (f(x,t) \ , \ \tilde{\varphi}_{J,\ k}(x)) \ . \tag{3.7}$$

Hence, problem (P) reduces to find the values of the expansion coefficients $c_{u_{J,\ k}}(t)$ of the approximate solution (3.4). The algorithm of obtaining this solution can be described in two stages; namely the *semi-discrete formulation* and the *full discrete formulation*.

3.1. Semi Discrete Formulation

In this subsection space discretization is made by substituting (3.4) and (3.5) into (3.1), so we get

$$\sum_{k=0}^{2^J-1} \frac{dc_{u_{J,\,k}}(t)}{dt}\, \tilde{\varphi}_{J,\,k}(x) + \alpha \sum_{k=0}^{2^J-1} c_{u_{J,\,k}}(t)\, \frac{d\tilde{\varphi}_{J,\,k}(x)}{dx}$$

$$+ \beta \sum_{k=0}^{2^J-1} \sum_{l=0}^{2^J-1} c_{u_{J,\,k}}(t)\, c_{u_{J,\,l}}(t)\, \tilde{\varphi}_{J,\,k}(x)\, \frac{d\tilde{\varphi}_{J,\,l}(x)}{dx}$$

$$- \gamma \sum_{k=0}^{2^J-1} \frac{dc_{u_{J,\,k}}(t)}{dt}\, \frac{d^2\tilde{\varphi}_{J,\,k}(x)}{dx^2} = \sum_{k=0}^{2^J-1} c_{f_{J,\,k}}(t)\, \tilde{\varphi}_{J,\,k}(x). \quad (3.8)$$

Using $\tilde{\varphi}_{J,\,m}(x)$ as a test function in (3.8) we get

$$\sum_{k=0}^{2^J-1} \frac{dc_{u_{J,\,k}}(t)}{dt}\, (\tilde{\varphi}_{J,\,k}(x)\ ,\ \tilde{\varphi}_{J,\,m}(x))$$

$$+ \alpha \sum_{k=0}^{2^J-1} c_{u_{J,\,k}}(t)\, (\frac{d\tilde{\varphi}_{J,\,k}(x)}{dx}\ ,\ \tilde{\varphi}_{J,\,m}(x))$$

$$+ \beta \sum_{k=0}^{2^J-1} \sum_{l=0}^{2^J-1} c_{u_{J,\,k}}(t)\, c_{u_{J,\,l}}(t)\, (\tilde{\varphi}_{J,\,k}(x)\, \frac{d\tilde{\varphi}_{J,\,l}(x)}{dx}\ ,\ \tilde{\varphi}_{J,\,m}(x))$$

$$- \gamma \sum_{k=0}^{2^J-1} \frac{dc_{u_{J,\,k}}(t)}{dt}\, (\frac{d^2\tilde{\varphi}_{J,\,k}(x)}{dx^2}\ ,\ \tilde{\varphi}_{J,\,m}(x))$$

$$= \sum_{k=0}^{2^J-1} c_{f_{J,\,k}}(t)\, (\tilde{\varphi}_{J,\,k}(x)\ ,\ \tilde{\varphi}_{J,\,m}(x)). \quad (3.9)$$

Using orthonormality of the periodized scaling functions we obtain

$$\frac{dc_{u_{J,\,m}}(t)}{dt} + \alpha \sum_{k=0}^{2^J-1} c_{u_{J,\,k}}(t)\, \Gamma^1_{J,\,k,\,m}$$
$$+ \beta \sum_{k=0}^{2^J-1} \sum_{l=0}^{2^J-1} c_{u_{J,\,k}}(t)\, c_{u_{J,\,l}}(t)\, \Lambda^{0,1}_{J,\,k,\,l,\,m}$$
$$-\ \gamma \sum_{k=0}^{2^J-1} \frac{dc_{u_{J,\,k}}(t)}{dt}\, \Gamma^2_{J,\,k,\,m} \ = \ c_{f_{J,\,m}}(t)\ , \quad m = 0,\ 1,\ ...,\ 2^J\text{-}1, \quad (3.10)$$

where the *two-termconnection coefficients* $\Gamma^2_{J,\,k,\,m}$ and $\Gamma^1_{J,\,k,\,m}$ are given by

$$\Gamma^p_{J,\,k,\,m} = (\frac{d^p\tilde{\varphi}_{J,\,k}(x)}{dx^p}\ ,\ \tilde{\varphi}_{J,\,m}(x)), \quad p = 1,\ 2, \tag{3.11}$$

and the *three-termconnection coefficients* $\Lambda^{0,1}_{J,\,k,\,l,\,m}$ are given by

$$\Lambda^{d_1,\,d_2}_{J,\,k,\,l,\,m} = (\frac{d^{p_1}\tilde{\varphi}_{J,\,k}(x)}{dx^{p_1}}\ \frac{d^{p_2}\tilde{\varphi}_{J,\,l}(x)}{dx^{p_2}}\ ,\ \tilde{\varphi}_{J,\,m}(x)), \quad p_1 = 0,\ p_2 = 1\,. \tag{3.12}$$

The computation of these connection coefficients is given in details in [2, 4, 16]. We must notice that, (3.10) is a system of nonlinear ordinary differential equations, in the unknowns $c_{u_{J,\,k}}(t)$, which can be written in matrix form as follow:

$$(\mathbf{I} - \mathbf{N})\ \dot{\mathbf{C}}_u + \mathbf{G}_b\ \mathbf{C}_u + \mathbf{E}_b\ \bar{\mathbf{C}}_u = \mathbf{C}_{fb}, \tag{3.13}$$

$$u(x,0) = u_0(x), \tag{3.14}$$

where $c_{u_l} = c_{u_{J,\,l}}$ and $c_{f\,l} = c_{f_{J,\,l}}$ for a fixed J and

$$\dot{c}_{u_m} = \frac{dc_{u_{J,\,m}}}{dt}, \tag{3.15a}$$

$$c_{u_m} = (u(x,t),\ \tilde{\varphi}_{J,\,m}(x)), \tag{3.15b}$$

$$c_{fb_m} = (f(x,t),\ \tilde{\varphi}_{J,\ m}(x)), \tag{3.15c}$$

are the entries of the 2^J length vectors $\dot{\mathbf{C}}_u$, $\mathbf{C}_u$ and $\mathbf{C}_{fb}$ respectively and the entries of the 2^{2J} length vector $\bar{\mathbf{C}}_u$ are

$$\bar{c}_{u_{k,\ l}} = c_{u_{J,\ k}}\ c_{u_{J,\ l}}\,. \tag{3.16}$$

Also, the entries of the ($2^J \times 2^J$)-square matrix $\mathbf{G}_b = [g_{b_{m,\ k}}]$, are given by

$$g_{b_{m,\ k}} = \alpha\ \Gamma^1_{J,\ k,\ m}\ , \tag{3.17}$$

and the entries of the ($2^J \times 2^{2J}$)-rectangular matrix $\mathbf{E}_b = [e_{b_{m,\ k,\ l}}]$, are given by

$$e_{b_{m,\ k,\ l}} = \beta\ \Lambda^{0,\ 1}_{J,\ k,\ l,\ m}\ , \tag{3.18}$$

Also, the entries of the ($2^J \times 2^J$)-square matrix $\mathbf{N} = [n_{m,\ k}]$, are given by

$$n_{m,\ k} = -\gamma\ \Gamma^2_{J,\ k,\ m}\ , \tag{3.19}$$

So, equation (3.13) is written in the form

$$\dot{\mathbf{C}}_u + \mathbf{G}\ \mathbf{C}_u + \mathbf{E}\ \bar{\mathbf{C}}_u = \mathbf{C}_f\ , \tag{3.20}$$

where

$$\mathbf{G} = (\mathbf{I} - \mathbf{N})^{-1}\ \mathbf{G}_b\,, \tag{3.21}$$

$$\mathbf{E} = (\mathbf{I} - \mathbf{N})^{-1}\ \mathbf{E}_b\,, \tag{3.22}$$

$$\mathbf{C}_f = (\mathbf{I} - \mathbf{N})^{-1}\ \mathbf{C}_{fb}\,. \tag{3.23}$$

3.2. Full Discrete Formulation

In this subsection the time discretization of the semi discrete scheme (3.20) is made using the backward difference [6, 7] at which (3.20) is evaluated at the forward end of the time step $t_n = n\Delta t$, Δt = time step length. Thus, we have

$$\dot{\mathbf{C}}_u^n + \mathbf{G}\ \mathbf{C}_u^n + \mathbf{E}\ \bar{\mathbf{C}}_u^n = \mathbf{C}_f^n\,. \tag{3.24}$$

The time derivative is then approximated by a backward difference over the time step:

$$\dot{\mathbf{C}}_u^n = \frac{\mathbf{C}_u^n - \mathbf{C}_u^{n-1}}{\Delta t}\,. \tag{3.25}$$

The full discretization is done by substituting (3.25) into (3.24). So, we get

$$\mathbf{A}\ \mathbf{C}_u^n + \mathbf{Q}\ \bar{\mathbf{C}}_u^n = \mathbf{b}^n\ , \tag{3.26}$$

where

$$\mathbf{A} = \ \mathbf{I} + \Delta t\ \mathbf{G}\ , \tag{3.27}$$

$$\mathbf{Q} = \ \Delta t\ \mathbf{E}\ , \tag{3.28}$$

$$\mathbf{b}^n = \ \Delta t\ \mathbf{C}_f^n \ + \ \mathbf{C}_u^{n-1}\ , \tag{3.29}$$

$$c_{u_m}^0 = c_{u_{J,\,m}}(0) = (u_0(x)\ ,\ \tilde{\varphi}_{J,\,m}(x))\ , \tag{3.30}$$

$$\bar{c}_{u_{k,\,l}}^0 = c_{u_k}^0\ c_{u_l}^0, \tag{3.31}$$

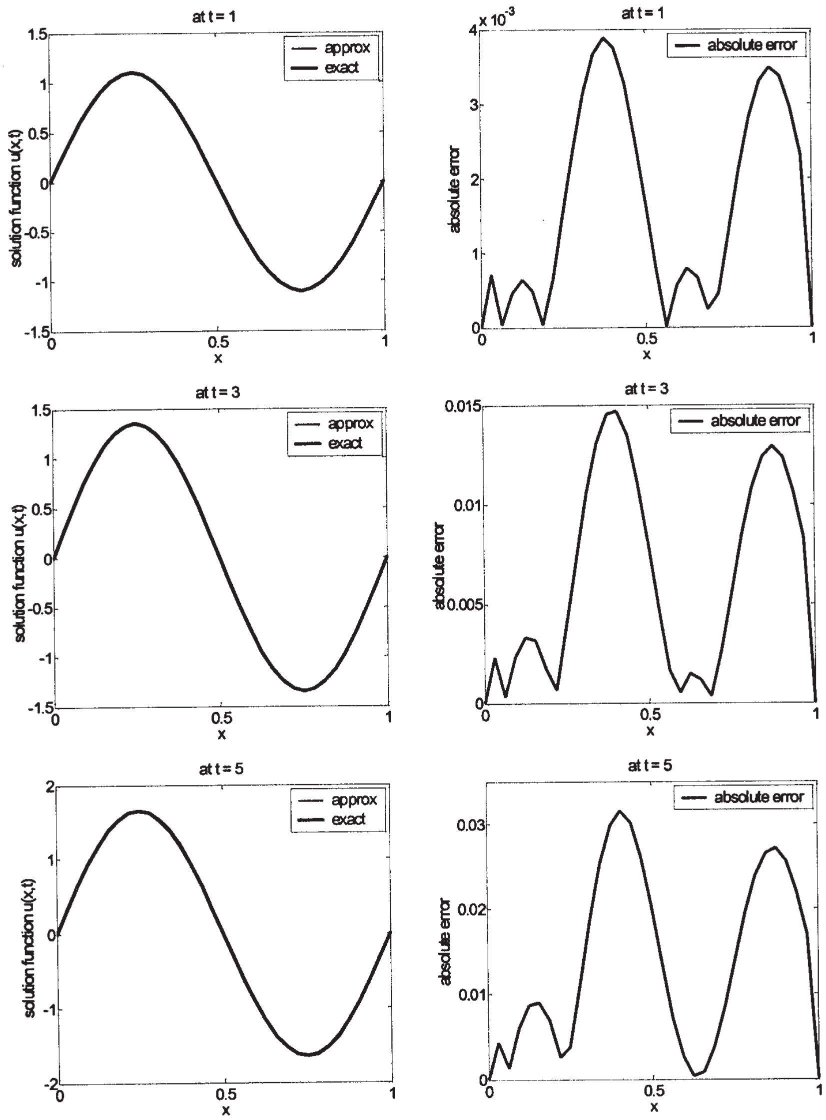

Figure 1: Comparison of the wavelet-Galerkin solution and exact solution and the corresponding absolute error for Example 1 at $t = 1$, at $t = 3$ and at $t = 5$

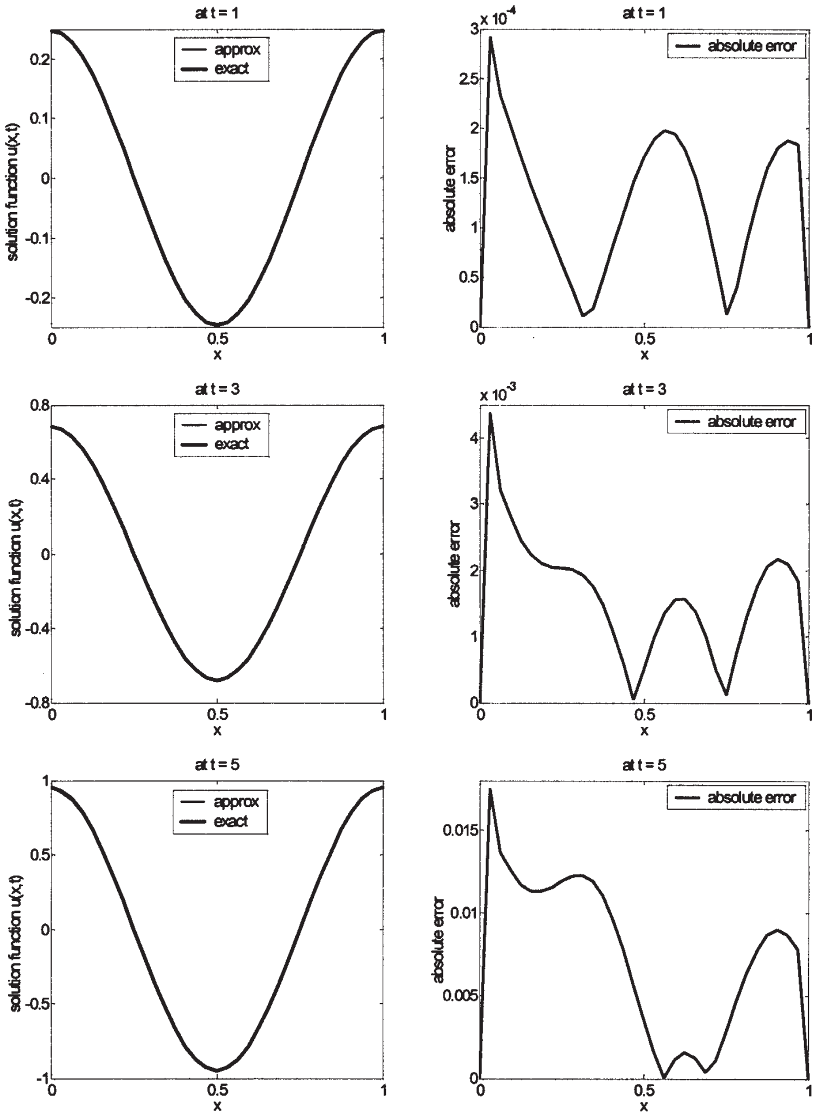

Figure 2: Comparison of the wavelet-Galerkin solution and exact solution and the corresponding absolute error for Example 2 at $t = 1$, at $t = 3$ and at $t = 5$

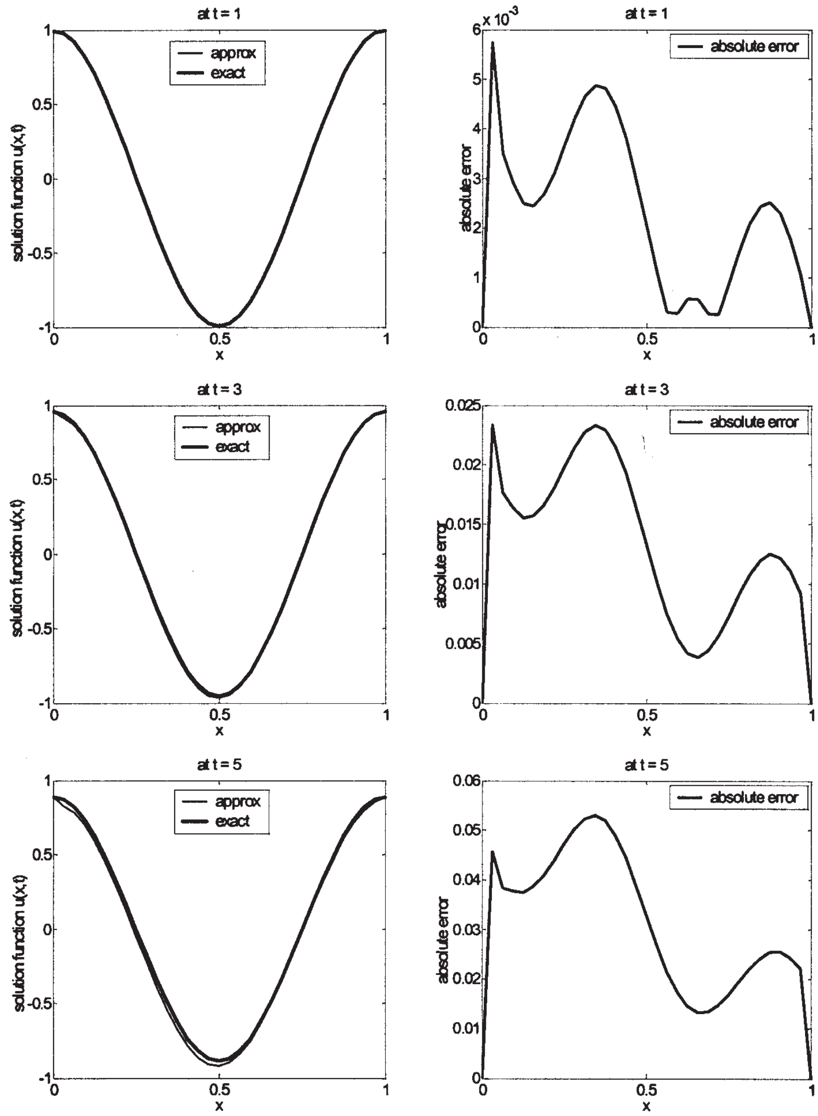

Figure 3: Comparison of the wavelet-Galerkin solution and exact solution and the corresponding absolute error for Example 3 at $t = 1$, at $t = 3$ and at $t = 5$

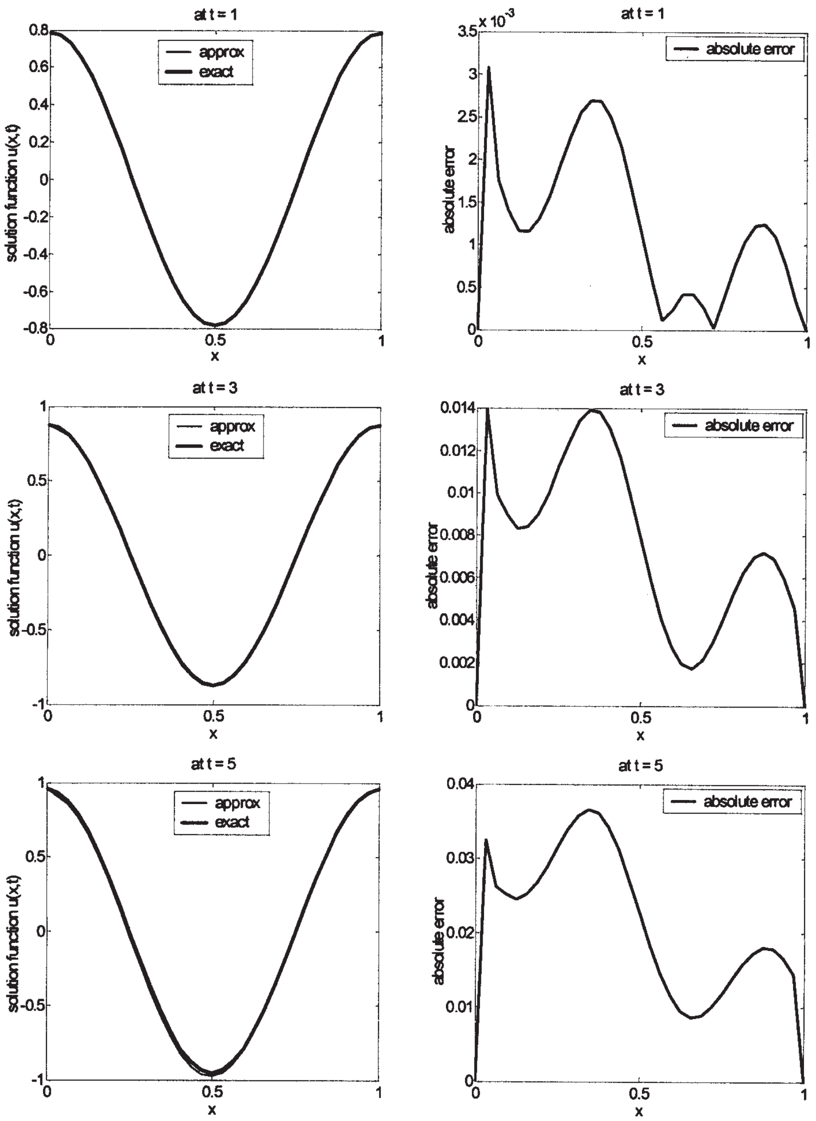

Figure 4: Comparison of the wavelet-Galerkin solution and exact solution and the corresponding absolute error for Example 4 at $t = 1$, at $t = 3$ and at $t = 5$

which is a system of nonlinear algebraic equations. This completes the full discretization and we get the full discrete wavelet-Galerkin formulation. Solving the system (3.26) using Newton's method [6, 18] we get the expansion coefficients $c_{u_{J,\,k}}$ and substituting in (3.4) we obtain the approximate solution $u_J(x,t)$.

4. Numerical Experiment

The proposed Wavelet-Galerkin method is used to solve the periodic BBM equation defined in problem (P) with $\alpha = 1$, $\beta = 1$ and $\gamma = 1$. The comparison of the Wavelet-Galerkin solution with the exact one in different examples is given in what follows. Here we use $\Delta t = 0.05$, and $J = 5$ with $D = 8$.

Example 1. In this example we have the right hand side function, the boundary conditions and the initial condition values as follows:

$$f(x,t) = ((0.1 + 0.4\pi^2)\sin 2\pi x + 2\pi\cos 2\pi x)e^{0.1t} + \pi e^{0.2t}\ \sin 4\pi x,$$

$$u_b = 0, \quad u_0(x) = \sin 2\pi x.$$

The wavelet solution is compared with the exact solution which is

$$u(x,t) = e^{0.1t}\ \sin 2\pi x,$$

at times $t = 1$, $t = 3$ and $t = 5$ as shown in Figure 1. Also, the absolute error is shown in the figure.

Example 2. In this example we have the right hand side function, the boundary conditions and the initial condition values as follows:

$$f(x,t) = (0.25 + \pi^2)\cos 0.25t\cos 2\pi x - 2\pi\sin 0.25t\sin 2\pi x \\ - \pi\sin^2 0.25t\sin 4\pi x,$$

$$u_b = \sin 0.25t, \quad u_0(x) = 0.$$

The wavelet solution is compared with the exact solution which is

$$u(x,t) = \sin 0.25t \cos 2\pi x,$$

at times $t = 1$, $t = 3$ and $t = 5$ as shown in Figure 2. Also, the absolute error is shown in the figure.

Example 3. In this example we have the right hand side function, the boundary conditions and the initial condition values as follows:

$$f(x,t) = -(0.1 + 0.4\pi^2) \text{ sech } 0.1t \ \tanh 0.1t \ \cos 2\pi x$$
$$- 2\pi \text{ sech } 0.1t \ \sin 2\pi x - \pi \text{ sech}^2 0.1t \ \sin 4\pi x,$$

$$u_b = \text{sech } 0.1t, \quad u_0(x) = \cos 2\pi x.$$

The wavelet solution is compared with the exact solution, which is

$$u(x,t) = \text{sech } 0.1t \cos 2\pi x,$$

at times $t = 1$, $t = 3$ and $t = 5$ as shown in Figure 3. Also, the absolute error is shown in the figure.

Example 4. In this example we have the right hand side function, the boundary conditions and the initial condition values as follows:

$$f(x,t) = \frac{0.1 + 0.4\pi^2}{0.1t + \frac{2\pi}{3}} \cos 2\pi x \ - 2\pi \ \ln \ (0.1 \ t + \frac{2\pi}{3}) \ \sin 2\pi x$$
$$-\pi \ (\ln \ (0.1 \ t + \frac{2\pi}{3}))^2 \ \sin 4\pi x,$$

$$u_b = \ln \ (0.1 \ t + \frac{2\pi}{3}), \quad u_0(x) = \ln \frac{2\pi}{3} \ \cos \ 2\pi x.$$

The wavelet solution is compared with the exact solution, which is

$$u(x,t) = \ln \ (0.1 \ t + \frac{2\pi}{3}) \cos 2\pi x,$$

at times $t = 1$, $t = 3$ and $t = 5$ as shown in Figure 2. Also, the absolute error is shown in the figure.

References

[1] K. Amaratunga, J.R. Williams, S. Qian, J. Weiss, Wavelet-Galerkin solutions for one dimensional partial differential equations, *Int. J. for Num. Methods in Engineering*, **37** (1994), 2703-2716.

[2] M.A. Amer, M.S. El-Azab, A.A. Ashour, W.S. El-Beshbeshy, Wavelet-Galerkin solution of periodic fourth order partial differential equations, *Applied Mathematics and Computation Journal*, Submited.

[3] M.A. Amer, M.S. El-Azab, A.A. Ashour, W.S. El-Beshbeshy, Wavelet-Galerkin solution of nonlinear nonhomogeneous Fisher equation, *Ain Shams Journal*, Submited.

[4] M.A. Amer, M.S. El-Azab, A.A. Ashour, W.S. El-Beshbeshy, Wavelet-Galerkin solution of nonhomogeneous Burgers equation, To Appear.

[5] A. Avudainayagam, C. Vani, Wavelet-Galerkin method for integro-differential equations, *Applied Numerical Mathematics*, **32** (2000), 247-254.

[6] R.L. Burden, J.D. Fairs, *Numerical Analysis*, Pws-Kent (1989).

[7] D.S. Burnentt, *Finite Element Analysis from Concepts to Applications*, Addisson-Wesley, California (1987).

[8] N.M. Chuong, B.K. Cuong, Galerkin-wavelet approximation for a class of partial integro-differential equations, *Fractional Calculus and Applied Analysis*, **4**, No. 2 (2001) 143-152.

[9] I. Daubechies, Orthonormal basis of compactly supported wavelets, *Comm. Pure and Appl. Math.*, **41** (1988), 909-996.

[10] I. Daubechies, *Ten Lectures on Wavelets*, SIAM (1992).

[11] L. Debnath, *Nonlinear Partial Differential Equations for Scientists and Engineers*, Birkhäuser, Boston (1997).

[12] P. Frick, A. Grossmann, P. Tohamitchain, Wavelet analysis of signals with gaps, *J. of Mathematical Physics*, **39**, No. 8 (1998), 4091-4107.

[13] J.C. Goswami, A.K. Chan, *Fundamentals of Wavelets: Theory, Algorithms, and Applications*, John Wiley and Sons, New York (1999).

[14] S.L. Ho, S.Y. Yang, Wavelet-Galerkin method for solving parabolic equations in finite domains, *Finite Elements in Analysis and Design*, **37** (2001), 1023-1037.

[15] S. Jianhua, Y. Xuming, Y. Biquan, S. Yuantong, Wavelet-Galerkin solutions for differential equations, *Wuhan University J. of Natural Sciences*, **3**, No. 4 (1998), 403-406.

[16] A. Latto, H.L. Resnikoff, E. Tenenbaum, The evaluation of connection coefficients of compactly supported wavelets, *Aware Technical Report*, AD910708 (1999).

[17] G. Schlossnagle, J.M. Restrepo, G.K. Leaf, Periodized wavelets, *Technical Report*, ANL-93/34 (1993).

[18] H.R. Shwarz, J. Waldvogle, *Numerical Analysis, A Comprehensive Introduction*, John Wiley and Sons, New York (1989).

[19] G. Strang, Wavelets and transforms versus fourier transforms, *American Mathematical Society*, **28**, No. 2, 288-305.

[20] J.R. Williams, K. Amaratunga, Introduction to wavelets in engineering, *Int. J. Numer. Methods in Engineering*, **37** (1994), 2365-2388.